

ADVANCED MATH'S 2

FORM SIX-JOINT

MARKING SCHEME

1 a) i) Given; $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$

Solution

$P(B) = y_1$ For Mutually exclusive

From $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ——— 0½ Marks

But $P(A \cap B) = 0$ For Mutually exclusive ——— 0½ Marks

$$P(A \cup B) = P(A) + P(B)$$

$$\frac{3}{5} = \frac{1}{2} + y_1$$

$$y_1 = \frac{1}{10}$$
 ——— 0½ Marks

But $P(B) = y_2$ For Independent event

From $P(A \cap B) = P(A) \times P(B)$ ——— 0½ Marks

But $P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

$$\frac{3}{5} = \frac{1}{2} + y_2 - \frac{y_2}{2}$$
 ——— 0½ Marks

$$\frac{1}{10} = \frac{y_2}{2}$$

$$y_2 = \frac{1}{5}$$

$$y_1 + y_2 = \frac{1}{10} + \frac{1}{5} = \frac{3}{10}$$

$$\therefore \text{Value of } y_1 + y_2 = \frac{3}{10}$$
 ——— 0½ Marks

ii) Given; Box with correction Pen = {1, 2, 3, 4, 5, 6, 7, 8 and 9}

Coin = {H, T} and Die = {1, 2, 3, 4, 5, 6}

Even number = {2, 4, 6, 8} From the Box

Prime number = {2, 3, 5, 7} From the Box

= Break the experiment into two different and work out the number of outcomes separately. Then add these results

First Experiment

Second experiment

[Even number) and (toss a coin)] or [Prime number) and (throw a die)] ——— 0½ Marks

$$(4 \times 2) +$$

$$(4 \times 6)$$

$$8 + 24 = 32$$

$\therefore$ 32 outcome are possible ——— 0½ Marks

1 (b) i) Father and Mother next to each other

$$= (n-1-1)! \times 2! \quad \text{--- 01 Marks}$$

$$= (n-2)! \times 2! \quad \text{But } n=6$$

$$= (6-2)! \times 2!$$

$$= 4! \times 2!$$

$$= 48 \text{ ways}$$

$$\therefore \text{Number of ways} = 48 \text{ ways} \quad \text{--- 01 Marks}$$

ii) Father and Mother not next to each other

$$\text{Not next} = \text{Total} - \text{Next} \quad \text{--- } 0\frac{1}{2} \text{ Marks}$$

$$\text{Total} = (n-1)! = (6-1)! = 5! \quad \text{--- } 0\frac{1}{2} \text{ Marks}$$

$$\text{Total} = 120 \text{ ways}$$

$$\text{Not next} = 120 - 48$$

$$= 72 \text{ ways}$$

$$\therefore \text{Number of ways} = 72 \text{ ways} \quad \text{--- 01 Marks}$$

$$c) \text{ Given, } p = 0.0001, n = 2000, \lambda = np$$

$$\lambda = 2000(0.0001) = 2$$

$$\text{At least 2 will suffer} = \{x = 2, 3, 4, \dots, 2000\} = (x \geq 2)$$

$$P(x \geq 2) = 1 - P(x \leq 1)$$

$$P(x \geq 2) = 1 - P(x = 1 \text{ or } 0)$$

$$P(x \geq 2) = 1 - [P(x=1) + P(x=0)]$$

$$\text{But } P(x=x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad \text{--- 01 Marks}$$

$$P(x \geq 2) = 1 - \left[\frac{\lambda^1 e^{-\lambda}}{1!} + \frac{\lambda^0 e^{-\lambda}}{0!} \right] \quad \text{--- 01 Marks}$$

$$= 1 - [2e^{-2} + e^{-2}]$$

$$= 1 - 3e^{-2}$$

$$P(x \geq 2) = 0.5940$$

$$\therefore \text{The probability is } 0.5940 \quad \text{--- 01 Marks}$$

$$2b)(ii) (P \wedge Q) \vee [(\neg r \wedge (Q \wedge P))] \dots \text{Given.}$$

$$(P \wedge Q) \vee [(\neg r \wedge (P \wedge Q))] \dots \text{Commutative law.}$$

$$((P \wedge Q) \wedge T) \vee [(\neg r \wedge (P \wedge Q))] \dots \text{Identity law.}$$

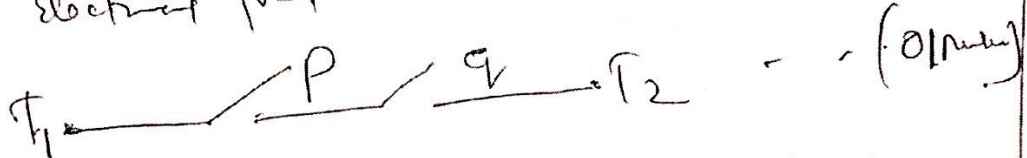
$$((P \wedge Q) \wedge T) \vee [(P \wedge Q) \wedge \neg r] \dots \text{Contradiction law.} \quad \text{--- (0/1 mark)}$$

$$(P \wedge Q) \wedge [T \vee \neg r] \dots \text{Distributive law.}$$

$$(P \wedge Q) \wedge T \dots \text{Identity law.} \quad \text{--- (0/1 mark)}$$

$$P \wedge Q \dots \text{Identity law.} \quad \text{--- (0/1 mark)}$$

The electric network.



C(i) Required to simplify the statement T.

Select the "F" in the T column.

$$(\neg P \vee G \vee \neg M) \wedge (\neg P \vee \neg G \vee M) \wedge (\neg P \vee \neg G \vee \neg M) \dots \text{Given.}$$

$$(\neg P \vee G \vee \neg M) \wedge [(\neg P \vee \neg G) \vee (M \wedge \neg M)] \dots \text{Distributive law.}$$

$$(\neg P \vee G \vee \neg M) \wedge [(\neg P \vee \neg G) \vee F] \dots \text{Complimentary law.}$$

$$(\neg P \vee G \vee \neg M) \wedge [(\neg P \vee \neg G) \vee F] \dots \text{Identity law.} \quad \text{--- (0/1 mark)}$$

$$(\neg P \vee G \vee \neg M) \wedge (\neg P \vee \neg G) \dots \text{Identity law.}$$

$$\neg P \vee [(G \vee \neg M) \wedge (\neg G)] \dots \text{Distributive law.}$$

(0/1 mark)

∴ 80.23% of students scored below Tom's score.
Therefore he scored, Tom is better than 70% of the
students who took the test. Tom will be admitted
to university. (10½ marks)

2 (a) (i) If Contrapositive you can comprehend geometry then -
you know how to reason deductively. . . . (10½ marks)

(ii) If Converse you cannot comprehend geometry then you
do not know how to reason deductively. . . . (10½ marks)

(ii) Contrapositive

If $a - b$ is not always a positive integer, then
two integers a and b are such that a is not q
times b (10½ marks)

Converse

If $a - b$ is always a positive integer, then two
integers a and b are such that a is not greater
than b (10½ marks)

(iii) Contrapositive

If a quadrilateral does not have two pairs of
parallel sides, then it is not a rectangle. . . . (10½ marks)

Converse

If a quadrilateral has two pairs of parallel sides,
then it is a rectangle. . . . (10½ marks)

(03)

2 (b)(i) Let $P: I$ work hard

Q: I get raise

r: I get a second job.

$$((p \rightarrow q) \wedge (q \rightarrow \sim r) \wedge r) \rightarrow \sim p \quad \text{--- (2) (3) (4) (5) (6) (7) (8) (9) (10) (11) (12) (13) (14) (15) (16) (17) (18) (19) (20) (21) (22) (23) (24) (25) (26) (27) (28) (29) (30) (31) (32) (33) (34) (35) (36) (37) (38) (39) (40) (41) (42) (43) (44) (45) (46) (47) (48) (49) (50) (51) (52) (53) (54) (55) (56) (57) (58) (59) (60) (61) (62) (63) (64) (65) (66) (67) (68) (69) (70) (71) (72) (73) (74) (75) (76) (77) (78) (79) (80) (81) (82) (83) (84) (85) (86) (87) (88) (89) (90) (91) (92) (93) (94) (95) (96) (97) (98) (99) (100) (101) (102) (103) (104) (105) (106) (107) (108) (109) (110) (111) (112) (113) (114) (115) (116) (117) (118) (119) (120) (121) (122) (123) (124) (125) (126) (127) (128) (129) (130) (131) (132) (133) (134) (135) (136) (137) (138) (139) (140) (141) (142) (143) (144) (145) (146) (147) (148) (149) (150) (151) (152) (153) (154) (155) (156) (157) (158) (159) (160) (161) (162) (163) (164) (165) (166) (167) (168) (169) (170) (171) (172) (173) (174) (175) (176) (177) (178) (179) (180) (181) (182) (183) (184) (185) (186) (187) (188) (189) (190) (191) (192) (193) (194) (195) (196) (197) (198) (199) (200) (201) (202) (203) (204) (205) (206) (207) (208) (209) (210) (211) (212) (213) (214) (215) (216) (217) (218) (219) (220) (221) (222) (223) (224) (225) (226) (227) (228) (229) (230) (231) (232) (233) (234) (235) (236) (237) (238) (239) (240) (241) (242) (243) (244) (245) (246) (247) (248) (249) (250) (251) (252) (253) (254) (255) (256) (257) (258) (259) (260) (261) (262) (263) (264) (265) (266) (267) (268) (269) (270) (271) (272) (273) (274) (275) (276) (277) (278) (279) (280) (281) (282) (283) (284) (285) (286) (287) (288) (289) (290) (291) (292) (293) (294) (295) (296) (297) (298) (299) (300) (301) (302) (303) (304) (305) (306) (307) (308) (309) (310) (311) (312) (313) (314) (315) (316) (317) (318) (319) (320) (321) (322) (323) (324) (325) (326) (327) (328) (329) (330) (331) (332) (333) (334) (335) (336) (337) (338) (339) (340) (341) (342) (343) (344) (345) (346) (347) (348) (349) (350) (351) (352) (353) (354) (355) (356) (357) (358) (359) (360) (361) (362) (363) (364) (365) (366) (367) (368) (369) (370) (371) (372) (373) (374) (375) (376) (377) (378) (379) (380) (381) (382) (383) (384) (385) (386) (387) (388) (389) (390) (391) (392) (393) (394) (395) (396) (397) (398) (399) (400) (401) (402) (403) (404) (405) (406) (407) (408) (409) (410) (411) (412) (413) (414) (415) (416) (417) (418) (419) (420) (421) (422) (423) (424) (425) (426) (427) (428) (429) (430) (431) (432) (433) (434) (435) (436) (437) (438) (439) (440) (441) (442) (443) (444) (445) (446) (447) (448) (449) (450) (451) (452) (453) (454) (455) (456) (457) (458) (459) (460) (461) (462) (463) (464) (465) (466) (467) (468) (469) (470) (471) (472) (473) (474) (475) (476) (477) (478) (479) (480) (481) (482) (483) (484) (485) (486) (487) (488) (489) (490) (491) (492) (493) (494) (495) (496) (497) (498) (499) (500) (501) (502) (503) (504) (505) (506) (507) (508) (509) (510) (511) (512) (513) (514) (515) (516) (517) (518) (519) (520) (521) (522) (523) (524) (525) (526) (527) (528) (529) (530) (531) (532) (533) (534) (535) (536) (537) (538) (539) (540) (541) (542) (543) (544) (545) (546) (547) (548) (549) (550) (551) (552) (553) (554) (555) (556) (557) (558) (559) (560) (561) (562) (563) (564) (565) (566) (567) (568) (569) (570) (571) (572) (573) (574) (575) (576) (577) (578) (579) (580) (581) (582) (583) (584) (585) (586) (587) (588) (589) (590) (591) (592) (593) (594) (595) (596) (597) (598) (599) (600) (601) (602) (603) (604) (605) (606) (607) (608) (609) (610) (611) (612) (613) (614) (615) (616) (617) (618) (619) (620) (621) (622) (623) (624) (625) (626) (627) (628) (629) (630) (631) (632) (633) (634) (635) (636) (637) (638) (639) (640) (641) (642) (643) (644) (645) (646) (647) (648) (649) (650) (651) (652) (653) (654) (655) (656) (657) (658) (659) (660) (661) (662) (663) (664) (665) (666) (667) (668) (669) (670) (671) (672) (673) (674) (675) (676) (677) (678) (679) (680) (681) (682) (683) (684) (685) (686) (687) (688) (689) (690) (691) (692) (693) (694) (695) (696) (697) (698) (699) (700) (701) (702) (703) (704) (705) (706) (707) (708) (709) (710) (711) (712) (713) (714) (715) (716) (717) (718) (719) (720) (721) (722) (723) (724) (725) (726) (727) (728) (729) (730) (731) (732) (733) (734) (735) (736) (737) (738) (739) (740) (741) (742) (743) (744) (745) (746) (747) (748) (749) (750) (751) (752) (753) (754) (755) (756) (757) (758) (759) (760) (761) (762) (763) (764) (765) (766) (767) (768) (769) (770) (771) (772) (773) (774) (775) (776) (777) (778) (779) (780) (781) (782) (783) (784) (785) (786) (787) (788) (789) (790) (791) (792) (793) (794) (795) (796) (797) (798) (799) (800) (801) (802) (803) (804) (805) (806) (807) (808) (809) (810) (811) (812) (813) (814) (815) (816) (817) (818) (819) (820) (821) (822) (823) (824) (825) (826) (827) (828) (829) (830) (831) (832) (833) (83$$

Then table of $((P \rightarrow Q) \wedge (Q \rightarrow \sim P)) \rightarrow \sim P$.

					A	B	C	
P	q	r	$\sim p$	$\sim r$	$p \rightarrow q$	$q \rightarrow \sim r$	$A \wedge B \wedge r$	Contr.
T	T	T	F	F	T	F	F	T
T	T	F	F	T	T	T	F	T
T	F	T	F	F	F	T	F	T
T	F	F	F	T	F	T	F	T
F	T	T	T	F	T	F	F	T
F	T	F	T	T	T	T	F	T
F	F	T	T	F	T	T	T	T
F	F	F	T	T	T	T	F	T

Since the final column has all 1's, therefore, the Argument is valid (odd # of 1's).

05

2 (i) ~~$\sim P \vee [(G \vee \sim M) \wedge \sim G]$~~ ... Distribution law.

2 (c)(i) Required to simplify the statement.
Select the 'T' in the column.

$$(P \wedge G \wedge M) \vee (P \wedge G \wedge \sim M) \vee (P \wedge \sim G \wedge \sim M) \vee (\sim P \wedge G \wedge M) \dots \text{Given}$$

$$[(P \wedge G) \wedge (M \vee \sim M)] \vee [(P \wedge \sim G) \wedge M] \vee (P \wedge \sim G \wedge \sim M) \vee [\sim P \wedge G \wedge M] \dots \text{Given}$$

$$(P \wedge G) \wedge (M \vee \sim M) \vee (P \wedge \sim G) \wedge (M \vee \sim M) \vee (\sim P \wedge G \wedge M) \dots \text{Distributive law}$$

$$[(P \wedge G) \wedge T] \vee [(P \wedge \sim G) \wedge T] \vee (\sim P \wedge G \wedge M) \dots \text{Complementary law}$$

$$(P \wedge G) \vee (P \wedge \sim G) \vee (\sim P \wedge G \wedge M) \dots \text{Identity law}$$

$$P \wedge (G \vee \sim G) \vee (\sim P \wedge G \wedge M) \dots \text{Distributive law}$$

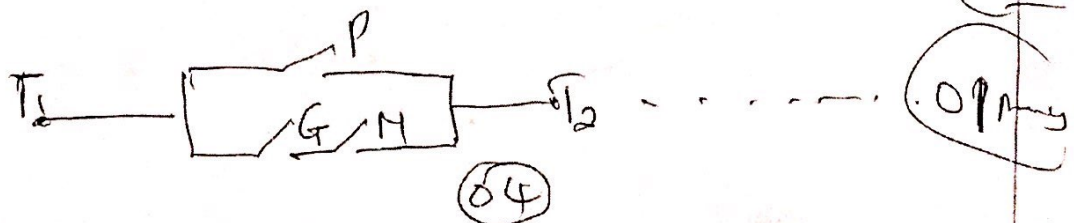
$$P \wedge T \vee (\sim P \wedge G \wedge M) \dots \text{Complementary law}$$

$$P \vee (\sim P \wedge G \wedge M) \dots \text{Identity law}$$

$$(P \vee \sim P) \wedge (P \vee (G \wedge M)) \dots \text{Distributive law}$$

$$T \wedge (P \vee (G \wedge M)) \dots \text{Complementary law}$$

$$P \vee (G \wedge M) \dots \text{Identity law}$$



3 (i) $f_m = 5 \text{ unit}$

$$\vec{r} = 2\hat{i} - 2\hat{j} + k$$

$$|\vec{r}| = \sqrt{2^2 + (-2)^2 + 1^2} = \sqrt{9} = 3 \dots \text{ (01 mark)}$$

$$F = f_m \left(\frac{\vec{r}}{|\vec{r}|} \right) = 5 \left(\frac{2\hat{i} - 2\hat{j} + k}{3} \right) = \frac{5}{3} (2\hat{i} - 2\hat{j} + k) \dots \text{ (01 mark)}$$

$$F = \frac{5}{3} (2\hat{i} - 2\hat{j} + k) \dots \text{ (1)} \dots \text{ (01 mark)}$$

No distance from $(5, 3, 7)$ to $(1, 2, 3)$

$$d = (5, 3, 7) - (1, 2, 3) = (4, 1, 4)$$

$$d = (4, 1, 4) \dots \text{ (11)} \dots \text{ (01 mark)}$$

Work done = $F \cdot d$

$$\text{Work done} = \frac{5}{3} \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 1 \\ 4 \end{pmatrix} = \frac{5}{3} (8 - 2 + 4) = \frac{50}{3}$$

$$\text{Work done} = \frac{50}{3} \text{ Joules} \dots \text{ (01 mark)}$$

(ii) $\phi(x, y, z) = x^2 y z$

$$\vec{a} = 3x^2 y \hat{i} + y z^2 \hat{j} - x z \hat{k}$$

$$(\phi_{\vec{a}}) = x^2 y z (3x^2 y \hat{i} + y z^2 \hat{j} - x z \hat{k}) \dots \text{ (01 mark)}$$

$$(\phi_{\vec{a}}) = 3x^4 y^2 z \hat{i} + x^2 y^2 z^3 \hat{j} - x^3 y z^2 \hat{k} \dots \text{ (01 mark)}$$

$$\frac{\partial}{\partial z} (\phi_{\vec{a}}) = 3x^4 y^2 \hat{i} + 3x^2 y^2 z^2 \hat{j} - 2x^3 y z \hat{k}$$

$$\frac{\partial^2}{\partial z^2} (\phi_{\vec{a}}) = 6x^4 y \hat{i} + 6x^2 y z \hat{j} - 2x^3 \hat{k} \dots \text{ (01 mark)}$$

(06)

at point $(1, -2, -1)$

$$\frac{\partial^2}{\partial y \partial z} (\phi_a) = 6(1)^4(-2)^2 + 6(1)^2(-2)^2(-1)^2 - 2(1)^3(-1)^2$$

$$\frac{\partial^2}{\partial y \partial z} (\phi_a) = 12i + 12j + 2k. \quad \text{--- OI Mark}$$

$$\frac{\partial^2}{\partial x \partial z} (\phi_a) = -12i - 12j + 2k \quad \text{--- OI Mark}$$

(b) ~~Q~~ from $(a+b)^2 = a^2 + 2ab + b^2$

we know $a^2 = |a|^2$, $b^2 = |b|^2$, --- OI Mark

$$a \cdot b = |a||b|\cos\theta$$

$$(a+b)^2 = |a|^2 + 2|a||b|\cos\theta + |b|^2$$

but $|a| = |b| = 1$

$$(a+b)^2 = 1 + 2\cos\theta + 1 \quad \text{--- OI Mark}$$

$$(a+b)^2 = 2 + 2\cos\theta$$

$$\frac{1}{2}(a+b)^2 = 1 + \cos\theta$$

$$\frac{1}{2}(a+b)^2 = 2\cos^2\frac{\theta}{2}$$

$$\frac{1}{4}(a+b)^2 = \cos^2\frac{\theta}{2}$$

$$\cos(\frac{\theta}{2}) = \frac{1}{2}(a+b). \quad \text{--- OI Mark}$$

$\therefore \frac{1}{2}(a+b) = \cos(\frac{\theta}{2})$ Hence Proved. --- OI Mark

Q7.

~~$x^2 + 2x + 1 = 0$~~
 ~~$x^2 + 2x + 1 = 0$~~
 ~~$x^2 + 2x + 1 = 0$~~

4 (Q) $x^2 + (-2\cos\theta)x + 1 = 0$ Compare with
 $ax^2 + bx + c = 0$

$a = 1, b = -2\cos\theta, c = 1$

$$x = \frac{2\cos\theta \pm \sqrt{(-2\cos\theta)^2 - 4 \times 1 \times 1}}{2 \times 1} \quad \leftarrow \text{01 Nov}$$

$$x = \frac{2\cos\theta \pm 2\sqrt{-(\sin^2\theta)}}{2}$$

$$x = \frac{2\cos\theta \pm 2i\sin\theta}{2} \quad \leftarrow \text{01 Nov}$$

$$x = \cos\theta \pm i\sin\theta$$

$\therefore x = \cos\theta + i\sin\theta$ or $x = \cos\theta - i\sin\theta$ 01 Nov

(09)

$$4(b) \text{ Given } z = \sin \frac{\pi}{5} + i(1 - \cos \frac{\pi}{5})$$

$$|Mod(z)| = \sqrt{\sin^2 \frac{\pi}{5} + (1 - \cos \frac{\pi}{5})^2} \quad \text{--- 0 | mod}$$

$$|Mod(z)| = \sqrt{\sin^2 \frac{\pi}{5} + \cos^2 \frac{\pi}{5} + 1 - 2\cos \frac{\pi}{5}}$$

$$|Mod(z)| = \sqrt{1 + 1 - 2\cos \frac{\pi}{5}} \quad \text{--- 0 | mod}$$

$$|Mod(z)| = \sqrt{2(1 - \cos \frac{\pi}{5})}$$

$$|Mod(z)| = \sqrt{2(2\sin^2 \frac{\pi}{10})}$$

$$|Mod(z)| = 2\sin \frac{\pi}{10} \quad \text{--- 0 | mod}$$

$$Arg(z) = \tan^{-1} \left(\frac{1 - \cos \frac{\pi}{5}}{\sin \frac{\pi}{5}} \right)$$

$$Arg(z) = \tan^{-1} \left(\frac{2\sin^2 \frac{\pi}{10}}{2\sin \frac{\pi}{10} \cos \frac{\pi}{10}} \right) \quad \text{--- 0 | mod}$$

$$Arg(z) = \tan^{-1}(\tan \frac{\pi}{10})$$

$$Arg(z) = \frac{\pi}{10}$$

$$\therefore Arg(z) = \frac{\pi}{10} \quad \text{--- 0 | mod}$$

(10)

$$4 \text{ (c) } \frac{(3+4i)(1+2i)}{1+3i} = q(1+i)$$

$$\frac{3(1+2i) + (4i)(1+2i)}{1+3i} = q(1+i) \quad \text{--- (O/P) rule}$$

$$\frac{3+10i-8}{1+3i} = q+qi$$

$$\frac{-5+10i(1-i)}{(1+3i)(1-i)} = q+qi$$

$$\frac{-5(1-i) + (10i)(1-i)}{10} = q+qi \quad \text{--- (O/P) rule}$$

$$-5+15i+10i+30 = 10(q+qi)$$

$$25+25i = 10q+10qi$$

$$10q = 25$$

$$q = \frac{25}{10} = \frac{5}{2}$$

from

$$\frac{(3+4i)(1+2i)}{1+3i} = \frac{5}{2}(1+i) \quad \text{--- (O/P) rule}$$

Compare argument both sides

$$\arg\left(\frac{(3+4i)(1+2i)}{(1+3i)}\right) = \arg\left(\frac{5}{2}(1+i)\right)$$

$$\arg(3+4i) + \arg(1+2i) - \arg(1+3i) = \arg\left(\frac{5}{2}(1+i)\right)$$

$$\tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}(2) - \tan^{-1}(3) = \tan^{-1}\left(\frac{5}{2}\right)$$

$$\tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}(2) - \tan^{-1}(3) = \frac{\pi}{4} \quad \text{--- (O/P) rule}$$

(11)

(d)

$$z = x + iy$$

$$\frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1} = \frac{(2x+1)+2yi}{(-y+1)+ix}$$

$$\frac{((2x+1)+2yi)((-y+1)-ix)}{((-y+1)+ix)((-y+1)-ix)}$$

$$\frac{2x+1(-y+1) - ix(2x+1)+2yi(-y+1) - 2i^2xy}{(-y+1)^2 + x^2}$$

$$\frac{(2x+1)(-y+1) + 2xy + i((-2x^2+x) + 2y(-y+1))}{(-y+1)^2 + x^2}$$

$$\frac{(2x+1)(-y+1) - i(2x^2+x) + i(-2y^2+2y) + 2xy}{(-y+1)^2 + x^2}$$

$$\frac{(2x+1)(-y+1) + 2xy}{(-y+1)^2 + x^2} + i \left(\frac{2y - 2y^2 - 2x^2 - x}{(-y+1)^2 + x^2} \right)$$

$$\text{Im} \left(\frac{2z+1}{iz+1} \right) = -2$$

$$\frac{2y - 2y^2 - 2x^2 - x}{(-y+1)^2 + x^2} = -2$$

$$2y - 2y^2 - 2x^2 - x = -2x^2 - 2y^2 + 4y - 2$$

$$2y - x = 4y - 2$$

$$x + 2y - 2 = 0$$

$$y = -\frac{1}{2}x + 1$$

∴ It is a straight line with slope of $-\frac{1}{2}$.

(12)

SECTION B

5. (a) Let $y = \alpha + \beta \Rightarrow \tan(\alpha + \beta) = \tan y$.

$$y = \tan^{-1} \left(\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \right) \quad \text{--- (1) Me}$$

$$y = \tan^{-1} \left(\frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \left(\frac{m}{m+1}\right)\left(\frac{1}{2m+1}\right)} \right)$$

$$y = \left(\frac{\frac{m(2m+1) + (m+1)}{(m+1)(2m+1)}}{1 - \frac{m}{(m+1)(2m+1)}} \right) \quad \text{--- (1) Me}$$

$$y = \tan^{-1} \left(\frac{\frac{m(2m+1) + (m+1)}{(m+1)(2m+1)}}{\frac{(m+1)(2m+1) - m}{(m+1)(2m+1)}} \right) \quad \text{--- (1) Me}$$

$$y = \tan^{-1} \left(\frac{m(2m+1) + m+1}{(m+1)(2m+1) - m} \right)$$

$$y = \tan^{-1} \left(\frac{2m^2 + 2m + 1}{2m^2 + 2m + 1} \right)$$

$$y = \tan^{-1}(1) = \frac{\pi}{4} \quad \text{--- (1) Me}$$

$$\alpha + \beta = \frac{\pi}{4}.$$

(13)

$$5) (b). 24 \cos\left(\frac{\pi}{3}x\right) + 10 \sin\left(\frac{\pi}{3}x\right) = 13.$$

Consider:

$$24 \cos\left(\frac{\pi}{3}x\right) + 10 \sin\left(\frac{\pi}{3}x\right) = R \cos\left(\frac{\pi}{3}x - \alpha\right)$$

$$24 \cos \frac{\pi}{3}x + 10 \sin \frac{\pi}{3}x = R \cos \frac{\pi}{3}x \cos \alpha + R \sin \frac{\pi}{3}x \sin \alpha$$

— 01 rule

By Comparing.

$$10 \sin\left(\frac{\pi}{3}x\right) = R \sin \alpha \sin \frac{\pi}{3}x.$$

$$R \sin \alpha = 10 \quad \text{--- (i)}$$

$$R \cos \alpha = 24 \quad \text{--- (ii)}$$

$$R^2 (\sin^2 \alpha + \cos^2 \alpha) = 24^2 + 10^2$$

$$R = \sqrt{676} = 26.$$

$$\tan \alpha = \frac{10}{24} \Rightarrow \alpha = \tan^{-1}\left(\frac{10}{24}\right)$$

— 01 rule

$$\alpha = \tan^{-1}\left(\frac{5}{12}\right)$$

$$26 \cos\left(\frac{\pi}{3}x - \tan^{-1}\left(\frac{5}{12}\right)\right) = 13.$$

$$\cos\left(\frac{\pi}{3}x - \tan^{-1}\left(\frac{5}{12}\right)\right) = \frac{1}{2}.$$

— 01 rule

$$\frac{\pi}{3}x - \tan^{-1}\left(\frac{5}{12}\right) = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\frac{\pi}{3}x - \tan^{-1}\left(\frac{5}{12}\right) = 2n\pi \pm \frac{\pi}{3}$$

$$x = \frac{6n\pi \pm 1}{\pi} + \frac{3}{\pi} \tan^{-1}\left(\frac{5}{12}\right).$$

— 01 rule

$$\therefore x = (6n \pm 1) + \frac{3}{\pi} \tan^{-1}\left(\frac{5}{12}\right).$$

14.

5) (i) Given that $\frac{1}{2\cos\theta + 5\sin\theta + 7}$.

Let $2\cos\theta + 5\sin\theta = R\cos(\theta - \alpha)$ --- 01 mark

$$2\cos\theta + 5\sin\theta = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$$

$$R\cos\alpha = 2 \quad \text{--- (i)}$$

$$R\sin\alpha = 5 \quad \text{--- (ii)} \quad \text{--- 01 mark}$$

$$R^2\cos^2\alpha + R^2\sin^2\alpha = 5^2 + 2^2$$

$$R^2 = 29$$

$$R = \sqrt{29}, \quad \alpha = \tan^{-1}\left(\frac{5}{2}\right) = \dots$$

--- 01 mark

$$\sqrt{29} \cos(\theta - 21.1^\circ)$$

Max. value $\sqrt{29} \cos(\theta - 21.1^\circ)$

$$\cos(\theta - 21.1^\circ) = 1$$

$$\therefore \text{max value} = \sqrt{29} \quad \text{--- 01 mark}$$

Min value $\sqrt{29} \cos(\theta - 21.1^\circ)$

$$\cos(\theta - 21.1^\circ) = -1$$

$$\therefore \text{Min value} = -\sqrt{29}$$

$$\frac{1}{\sqrt{29} + 7} = 0.0807 \quad \text{--- 01 mark}$$

$$\frac{1}{-\sqrt{29} + 7} = 1.6148$$

$\therefore$ The max. value is 1.6148 and min. value is 0.0807. --- 01 mark

(15)

5 (vii) Prove or L.H.S.

$$\frac{\tan x + \sec x + 1}{\tan x - \sec x + 1} = \frac{\sin x + 1 - \cos x}{\sin x - 1 + \cos x}$$

L.H.S.

$$\frac{\sin x + 1 - \cos x}{\sin x - 1 + \cos x} = \frac{2 \sin \frac{x}{2} \cos \frac{x}{2} + 1 - (1 - 2 \sin^2 \frac{x}{2})}{2 \sin \frac{x}{2} \cos \frac{x}{2} - 1 + (1 - 2 \sin^2 \frac{x}{2})}$$

(0/0) form

$$= \frac{\cos \frac{x}{2} + \sin \frac{x}{2}}{\cos \frac{x}{2} - \sin \frac{x}{2}} \left\{ \begin{array}{l} \text{Rationalize the} \\ \text{denominator} \end{array} \right. \quad \text{--- (0/0) form}$$

$$\frac{\cos \frac{x}{2} + \sin \frac{x}{2}}{\cos \frac{x}{2} - \sin \frac{x}{2}} \cdot \frac{(\cos \frac{x}{2} + \sin \frac{x}{2})}{(\cos \frac{x}{2} + \sin \frac{x}{2})} \quad \text{--- (0/0) form}$$

$$\frac{1 + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}$$

$$\frac{1 + \sin x}{\cos x}$$

--- (0/0) form

∴ Here prove that $\frac{\tan x + \sec x + 1}{\tan x - \sec x + 1} = \frac{1 + \sin x}{\cos x}$

--- (0/0) form

17

101

4) $z = x + iy$

$$\frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1} = \frac{(2x+1)+2yi}{(-y+1)+ix}$$

$$\frac{(2x+1)+2yi}{(-y+1)+ix} \cdot \frac{(-y+1)-ix}{(-y+1)-ix}$$

$$\frac{(2x+1)(-y+1) - i(2x+1) + 2yi(-y+1) - 2i^2xy}{(-y+1)^2 + x^2}$$

$$\frac{(2x+1)(-y+1) + 2xy + i(-2x^2+x) + 2yi(-y+1) - 2x^2}{(-y+1)^2 + x^2}$$

$$\frac{(2x+1)(-y+1) - i(2x^2+x) + i(-2y^2+2y) + 2xy}{(-y+1)^2 + x^2}$$

$$\frac{(2x+1)(-y+1) + 2xy}{(-y+1)^2 + x^2} + i \left(\frac{2y - 2y^2 - 2x^2 - x}{(-y+1)^2 + x^2} \right)$$

Im $\left(\frac{2z+1}{iz+1} \right) = -2$

$$\frac{2y - 2y^2 - 2x^2 - x}{(-y+1)^2 + x^2} = -2$$

$$2y - 2y^2 - 2x^2 - x = -2x^2 - 2y^2 + 4y - 2$$

$$2y - x = 4y - 2$$

$$x + 2y - 2 = 0$$

$$y = -\frac{1}{2}x + 1$$

∴ It is a straight line with slope of $-\frac{1}{2}$.

(12)

Q6 (i) Given $a^x = \left(\frac{a}{k}\right)^y$ { apply ln both side }

$$x \ln a = y \ln \left(\frac{a}{k}\right)$$

$$x \ln a = y \ln a - y \ln k$$

$$(x-y) \ln a = -y \ln k \quad \dots (i)$$

Also given $a^x = k^m$

$$x \ln a = m \ln k$$

$$\frac{x \ln a}{m} = \ln k \quad \dots (ii)$$

Substitute eqn (ii) into eqn (i)

$$(x-y) \ln a = -y \left(\frac{x \ln a}{m} \right)$$

$$x-y = -\frac{y x}{m}$$

$$mx - my = -xy$$

$$y = \frac{mx}{m-x}$$

Hence shown that $y = \frac{mx}{m-x}$

Q(1) Given $\frac{8x^2 - x - 1}{(x-1)^2}$

--- (i) ---

$$\text{Let } x-1 = u, x = u+1 \quad \dots (ii)$$

Substitute eqn (ii) into eqn (i)

$$\frac{8(u+1)^2 - (u+1) - 1}{u^2}$$

$$\frac{8u^2 + 16u + 6}{u^2} = 8 + \frac{16}{u} + \frac{6}{u^2} \quad (16)$$

Q (b)(ii) $2 \times 3 + 3 \times 4 + 4 \times 5 + \dots$

Separate the terms of the above series and terminate the general term.

$$2 + 3 + 4 + 5 + \dots + (n+1)$$

$$3 + 4 + 5 + 6 + \dots + (n+2)$$

Combine the above series.

$$2 \times 3 + 3 \times 4 + 4 \times 5 + \dots + (n+1)(n+2)$$

Represent a series in sigma notation.

$$2 \times 3 + 3 \times 4 + 4 \times 5 + \dots + (n+1)(n+2) = \sum_{r=1}^n (r+1)(r+2)$$

$$\sum_{r=1}^n (r+1)(r+2) = \sum_{r=1}^n (r^2 + 3r + 2) = \sum_{r=1}^n r^2 + 3 \sum_{r=1}^n r + \sum_{r=1}^n 2$$

$$= \frac{n}{6} (n+1)(2n+1) + \frac{3}{2} n(n+1) + 2n$$

$$= \frac{n}{6} (2n^2 + 12n + 22) = \frac{n}{3} (n^2 + 6n + 11)$$

$$\therefore \sum_{r=1}^n (r+1)(r+2) = \frac{n}{3} (n^2 + 6n + 11)$$

(18)

(10) Given the proposition.

(6)(i) $2^{n+2} + 3^{2n+1} = 7M$, where M is positive integer

Step 1, $n=1$, $2^{1+2} + 3^{2 \cdot 1 + 1} = 7M$... OK

$35 = 7M$, it is true for $n=1$... (i)

Assume $n=2$, $2^{2+2} + 3^{2(2)+1}$

$259 = 7M$... it is true for $n=2$... (ii)

Step 2: Assume it is true for $n=k$... OK

$2^{k+2} + 3^{2k+1} = 7M$,

$2^k = \frac{7M - 3^{2k+1}}{4}$

Step 3: We need to prove that it is true for $n=k+1$

$2^{k+3} + 3^{2k+3} = 2^3(2^k) + 3^3(3^{2k})$... OK
 $2^k = \frac{7M - 3^{2k+1}}{4}$ for eqn (ii)

$2^{k+3} + 3^{2k+3} = 2(7M - 3^{2k+1}) + 27(3^{2k})$... OK

$2^{k+3} + 3^{2k+3} = 14M - 6(3^{2k}) + 27(3^{2k})$... OK

$2^{k+3} + 3^{2k+3} = 14M + 21(3^{2k})$... OK
 $= 7(2M + 3(3^{2k}))$

$\therefore 2^{n+2} + 3^{2n+1}$ is divisible by 7. Proved

(ii) ~~Given~~ ^{sch} Define the General term of a series.

$2+3+4+5+\dots+(n+1) < 3+4+5+\dots+(n+2)$
 Include the General term to the given series.

$$\frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \dots + \frac{1}{(n+1)(n+2)} \quad - (0/1 \text{ mark})$$

Express in Sigma notation.

$$\frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \dots + \frac{1}{(n+1)(n+2)} = \sum_{r=1}^n \frac{1}{(r+1)(r+2)}$$

$$\text{Partial fraction } \frac{1}{(r+1)(r+2)} = \frac{A}{r+1} + \frac{B}{r+2} \quad - (0/1 \text{ mark})$$

$$1 = A(r+2) + B(r+1)$$

Equate Coeff of like terms

$$2A + B = 1 \quad - (i)$$

$$A + B = 0 \quad - (ii)$$

Solve the further eqn.

$$A = 1 \text{ and } B = -1$$

$$\text{Thus } \frac{1}{(r+1)(r+2)} = \frac{1}{r+1} - \frac{1}{r+2}$$

$$\sum_{r=1}^n \frac{1}{(r+1)(r+2)} = \sum_{r=1}^n \left(\frac{1}{r+1} - \frac{1}{r+2} \right)$$

$$\text{When } r=1, \quad \frac{1}{2} - \frac{1}{3}$$

$$r=2, \quad \frac{1}{3} - \frac{1}{4}$$

$$r=3, \quad \frac{1}{4} - \frac{1}{5}$$

$$\vdots$$

$$r=n-2, \quad \frac{1}{n-1} - \frac{1}{n}$$

$$r=n-1, \quad \frac{1}{n} - \frac{1}{n+1}$$

$$r=n, \quad \frac{1}{n+1} - \frac{1}{n+2}$$

(19)

$$\sum_{r=1}^n \frac{1}{(r+1)(r)} = \frac{1}{2} - \frac{1}{n+2} \quad \text{---}$$

$$\sum_{r=1}^{\infty} \frac{1}{(r+1)(r)} = \frac{1}{2} \quad \text{---} \quad \text{01 Note}$$

7 (Q) Form the D.E. is

$$y = (A \cos x + B \sin x) e^{-x}$$

multiply by e^x both side = - - - 0/mk

$$y e^x = A \cos x + B \sin x$$

$$e^x y + e^x \frac{dy}{dx} = -A \sin x + B \cos x \quad - - - 0/mk$$

$$e^x y + e^x \frac{dy}{dx} + e^x \frac{dy}{dx} + e^x \frac{d^2 y}{dx^2} = -A \cos x - B \sin x \quad - - - 0/mk$$

$$e^x y + 2e^x \frac{dy}{dx} + e^x \frac{d^2 y}{dx^2} = -y e^x$$

$$\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 2 = 0 \quad - - - 0/mk$$

$$\therefore \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 2 = 0 \quad (20) \quad - - - 0/mk$$

(20)

$$7 \text{ (b)(i)} \quad \frac{dy}{dx} = \left(\frac{2x+y-1}{x-2} \right)^2 \quad \dots \text{--- (i)}$$

$$\text{Let } x = X+h, y = Y+k, \quad \frac{dy}{dx} = \frac{dY}{dX}$$

$$\frac{dY}{dX} = \left(\frac{2(X+h) + (Y+k) - 1}{X+h-2} \right)^2 \quad \dots \text{--- (i) mod}$$

$$\frac{dY}{dX} = \left(\frac{2X+Y+2h+k-1}{X+h-2} \right)^2 \quad \dots \text{--- (ii)}$$

$$\text{Let } 2h+k=1 \quad \left\{ \begin{array}{l} h=2, k=-3 \\ h-2=0 \end{array} \right.$$

$$\frac{dY}{dX} = \left(\frac{2X+Y}{X} \right)^2 \quad \dots \text{--- (iii)}$$

$$\text{Let } Y = UV, \quad \frac{dY}{dX} = U + X \frac{dU}{dX} \quad \dots \text{--- (iv)}$$

Substitute eqn (iv) in eqn (iii) $\dots \text{--- (i) mod}$

$$U + X \frac{dU}{dX} = \left(\frac{2X+UX}{X} \right)^2$$

$$U + X \frac{dU}{dX} = 4 + 4U + U^2$$

$$X \frac{dU}{dX} = U^2 + 3U + 4 \quad \dots \text{--- (i) mod}$$

$$\int \frac{dU}{U^2 + 3U + 4} = \int \frac{1}{X} dX$$

$$\frac{4}{7} \left(\frac{dU}{U^2 + 3U + 4} \right) = \int \frac{1}{X} dX \quad \dots \text{--- (i) mod}$$

$$\frac{4}{7\sqrt{7}} \theta = \ln X + C, \quad \text{but } \theta = \tan^{-1} \left(\frac{2}{\sqrt{7}} \left(U + \frac{3}{2} \right) \right)$$

(21)

$$7 \text{ (b)(i)} \frac{2}{\sqrt{7}} \tan^{-1} \left(\frac{2}{\sqrt{7}} \left(u + \frac{3}{2} \right) \right) = \ln x + C \text{ but } u = \frac{y}{x}$$

$$\frac{2}{\sqrt{7}} \tan^{-1} \left(\frac{2}{\sqrt{7}} \left[\frac{y}{x} + \frac{3}{2} \right] \right) = \ln x + C, \text{ but } x = x - 4 = x - 2$$

$u - k = u + k = u + 3$

$$\frac{2}{\sqrt{7}} \tan^{-1} \left(\frac{2}{\sqrt{7}} \left(\frac{y+3}{x-2} + \frac{3}{2} \right) \right) = \ln(x-2) + C$$

— — — O/Marks

$$(b)(ii) (1+x^2) \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} = 0$$

$$\text{Let } p = \frac{dy}{dx}, \frac{dp}{dx} = \frac{d^2 y}{dx^2} \text{ — — — O/Marks}$$

$$1+x^2 \frac{dp}{dx} = -2p = 0.$$

$$\int \frac{dp}{p} = - \int \frac{2x}{1+x^2} dx \text{ — — — O/Marks}$$

$$\ln p = \ln(1+x^2)^{-1} \text{ — — — O/Marks}$$

$$p = \frac{1}{1+x^2}$$

— — — O/Marks

$$\frac{dy}{dx} = \frac{1}{1+x^2} \Rightarrow dy = \int \frac{dx}{1+x^2}$$

$$\therefore y = \tan^{-1}(x) + C \text{ — — — O/Marks}$$

(22)

7(d) From Newton's law of cooling.

$$\frac{d\theta}{dt} \propto (\theta - \theta_R)$$

$$\frac{d\theta}{dt} = k(\theta - \theta_R) \quad \text{--- o/n rule}$$

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta - \theta_R} = \int_0^t k dt$$

$$\ln[\theta - \theta_R] \Big|_{\theta_1}^{\theta_2} = [kt]_0^t \quad \text{--- o/n rule}$$

$$\ln(\theta_2 - \theta_R) - \ln(\theta_1 - \theta_R) = kt$$

$$\ln \left(\frac{\theta_2 - \theta_R}{\theta_1 - \theta_R} \right) = kt$$

$$\frac{\theta_2 - \theta_R}{\theta_1 - \theta_R} = e^{kt} \quad \text{--- o/n rule}$$

$$\theta_2 - \theta_R = (\theta_1 - \theta_R) e^{kt}$$

$$\theta_2 = \theta_R + (\theta_1 - \theta_R) e^{kt}$$

But $\theta_2 = T$ temp after time t .

$\theta_R = A$ Surrounding temp

$\theta_1 - \theta_R = B$ temp diff

$$\therefore T = A + B e^{kt}$$

(23)



(a)(i) Directrix

Is a fixed line whose distance from the focus is fixed. - (0/1 mark)

(ii) Latus Rectum

Is a focal chord perpendicular to the axis and parallel to the directrix. - (0/1 mark)

(iii) Focal chord

Is a line passing through focus and joining two points of the curve. - (0/1 mark)

(iv) Eccentricity.

Is a constant ratio of the distance of the variable point from the focus to the distance of the variable point from the directrix. - (0/1 mark)

(b)(i) $(y+5x)(y-5x) = 0$, $y^2 - x = 0$.

$y^2 = x$, the value of $a = \frac{1}{4}$. - (0/1 mark)

$y^2 = 4ax \Rightarrow a = \frac{1}{4}$

Vertex $(0,0)$

Focus $S(a,0) \Rightarrow S(\frac{1}{4},0)$

Equation of directrix $x = -a$, $x = -\frac{1}{4}$. - (0/1 mark)

(ii) $x^2 - 4y + 12 = 0$

$x^2 = 4y - 12$. - (0/1 mark)

$x^2 = 4(y-3)$ by comparing with the standard equation $(x-h)^2 = 4b(y-k)$, $h=0$, $k=3$, $b=1$. - (0/1 mark)

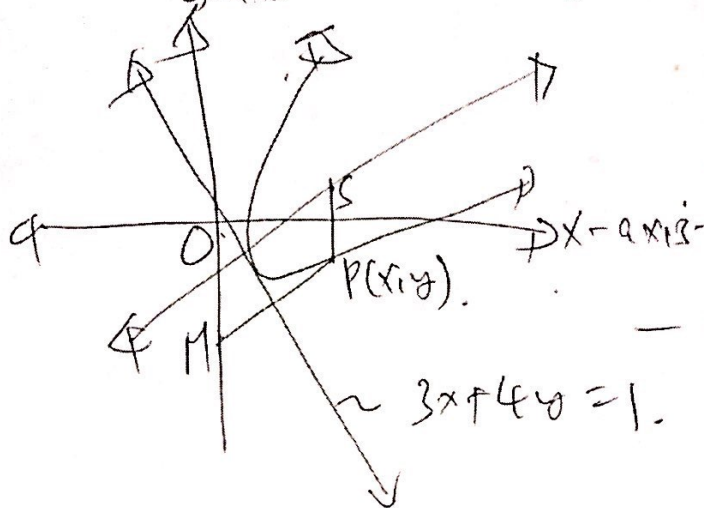
(24)

Vertex $V(h, k) = V(0, 3)$

Focus $S(h, b+k) = S(0, 4)$

Equation of directrix $y = -b+k, y = 2$
 - 01 mark

C (P) Consider the Diagram



From the figure above $\frac{SP}{MP} = e$, but $e = 1$
 - 01 mark

$$SP = MP$$

$$\sqrt{(x-3)^2 + y^2} = \left| \frac{3x + 4y - 1}{\sqrt{3^2 + 4^2}} \right|$$

- 01 mark

Sq. both sides.

$$(x-3)^2 + y^2 = \frac{(3x + 4y - 1)^2}{25}$$

$$25[x^2 + y^2 - 6x + 9] = 9x^2 + 16y^2 + 24xy - 6x - 8y + 1$$

- 01 mark

$$16x^2 + 9y^2 - 144x + 8y - 24xy + 224 = 0$$

∴ The equation of the parabola is

$$16x^2 + 9y^2 - 144x + 8y - 24xy + 224 = 0$$

- 01 mark

(25)

8(c)(ii) Eqn of parabola is

$$x - ty + at^2 = 0$$

Given $y^2 = 4x$, $a = 1$

$$x - ty + t^2 = 0$$

Alternative

$$y^2 = 4x, \quad 2y \frac{dy}{dx} = 4$$

$$\frac{dy}{dx} = \frac{2}{y}; \text{ Let } \frac{dy}{dx} = M$$

Slope of Tangent at $(t^2, 2t)$; $M = \frac{2}{2t} = \frac{1}{t}$

Eqn of Tangent, $y - y_0 = M(x - x_0) + y_0$

$$y = \frac{1}{t}(x - t^2) + 2t$$

The Eqn of the Tangent is $x - ty + t^2 = 0$

(d) Given $\frac{1}{4}(x^2 + y^2 - 4y)^2 = x^2 + y^2$

$$(x^2 + y^2 - 4y)^2 = 4(x^2 + y^2)$$

$$x^2 + y^2 - 4y = 2\sqrt{x^2 + y^2}$$

$$x^2 + y^2 = 4y + 2\sqrt{x^2 + y^2}$$

but $x^2 + y^2 = r^2$, $r = \sqrt{x^2 + y^2}$

$$r^2 = 4y + 2r \Rightarrow y = r \sin \theta$$

$$r^2 = 4r \sin \theta + 2r$$

$$r = 4 \sin \theta + 2$$

$\therefore r = 4 \sin \theta + 2$

(26)

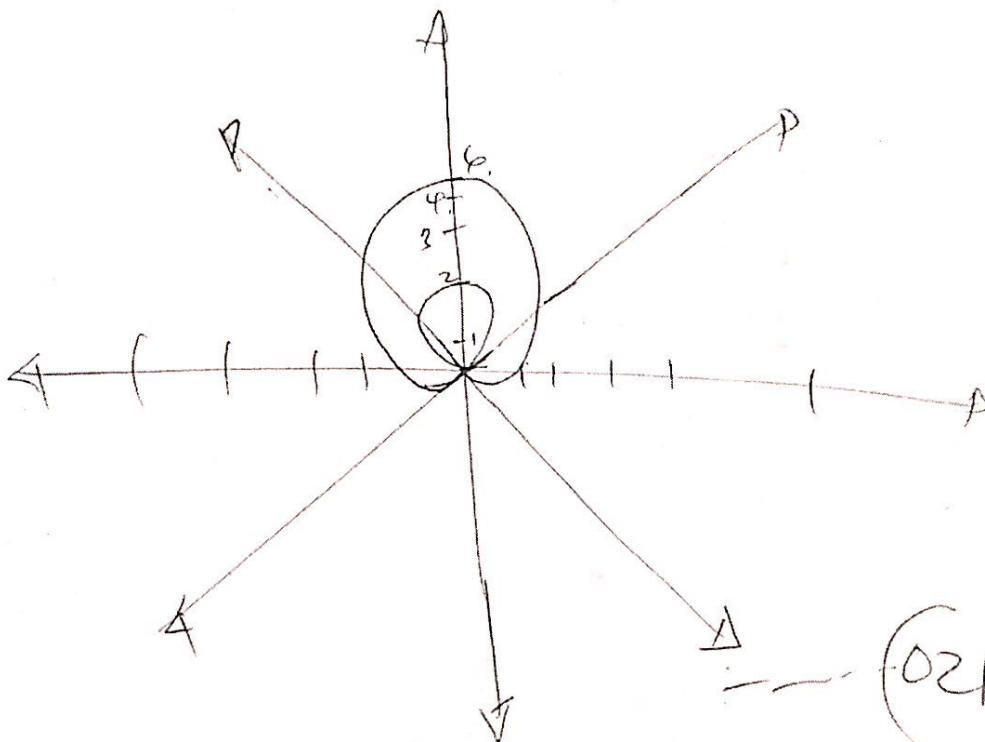
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Consider the Table below

θ	0°	30	60	90	120	150	180	210	240	270	300	330
$r = 4 \sin \theta + 2$	2	4	$2\sqrt{3}+2$	6	5.46	5.06	2	0	-1.46	-2	-1.46	2

--- (01 Mark)

Consider the Station below.



--- (02 Mark)

(27)