

DAR ES SALAAM CATHOLIC SECONDARY SCHOOLS ASSOCIATION (DACASSA)
JOINT EXAMINATIONS FOR CATHOLIC SCHOOLS (JECAS)
FORM FIVE ANNUAL EXAMINATIONS
ADVANCED MATHEMATICS

142

Time: 3:00 Hours

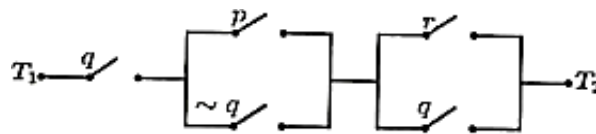
Year : 2026

Instructions

1. This paper consists of **ten (10)** questions, each carrying **ten (10)** marks.
2. Answer **all** questions.
3. All work done in answering each question must be shown clearly.
4. Non- programmable calculators may be used.
5. Cellular phones and any unauthorized materials are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s) provided.

1. (a) Simplify $(A - B) \cap (B - A)$ using laws of algebra / (Algebraic laws)
- (b) Shade set $B' \cap (C - A)$ on venn diagram
- (c) Each student in a class of 40 visits at least one of the following place during the holiday, Serengeti, Ngorongoro or Manyara ranch. 18 visits Serengeti, 20 visits Manyara ranch and 27 visits Ngorongoro. 7 visits Serengeti and Manyara ranch, 12 visits Manyara ranch and Ngorongoro and 4 visits Serengeti, Ngorongoro and Manyara ranch. Find the number of students who visits:
- Serengeti and Ngorongoro
 - Serengeti, Ngorongoro but not Manyara ranch
2. (a) Given $f(x) = 3 - x$, $g(x) = x^2 - 19$ and $h(x) = x - 2$, $x \in \mathbb{R}$. If $f(x) = g \circ h(x)$, find the values of x .
- (b) Given the function $f(x) = \frac{1}{x^2}$. State:
- The greatest possible domain for which the function is defined
 - The corresponding range of the function
 - Whether the function is one-to-one or many-to-one
- (c) Given the functions $f(x) = \frac{1}{x}$ and $g(x) = \frac{x}{x+2}$, sketch in the same set of axes the graph of $f(x)$ and the graph of $g(x)$ indicating clearly in the sketch the equations of asymptotes and the coordinates of any intersections between each curve with the coordinate axes.
3. (a) Without using the truth table “ Prove that the proposition $[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$ is a tautology”

(b) Given the electrical network below



Write a compound statement and simplify by using laws of proposition of logic

- (c) Test the validity of the following argument $[(p \rightarrow q) \wedge (\sim q \rightarrow \sim r)] \rightarrow (r \rightarrow p)$
4. (a) (i) If one of the root of the equation $ax^2 + bx + c = 0$ is twice the other root show that
- $$2b^2 = 9ac$$
- (ii) Use the principle of mathematical induction to prove that $17n^3 + 103n$ is divisible by 6 for all integers n .

(b) Solve the system of equation by determinant method

$$\begin{cases} \frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4 \\ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1 \\ \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2 \end{cases}$$

(c) Resolve $\frac{16x}{x^2 - 16}$ into partial fraction

5. (a) A circle whose centre is at $(3, -5)$ has equation $x^2 + y^2 - 6x + ay = 15$ where a is a constant. Find the value of a and the radius of the circle.

(b) Find the angle between the two straight line represented by the equation $2x^2 - 3xy + y^2 + 9x - 4y - 5 = 0$

(c) Find the area of the triangle whose vertices are the points $P(a, b+c)$, $Q(a, b-c)$ and $R(-a, c)$.

6. Kiboi Milling Company has two godowns Mikanjuni and Donge with capacities of 7000 kg and 4000 kg respectively. The company is to supply rice to three markets, Kange, Pongwe and Amboni whose requirements are 4500kg, 3000kg and 3500kg respectively. The transport cost per kg between the markets and the godowns is given in the following table:

Market	From	
	Mikanjuni	Donge
Kange	700/=	300/=
Pongwe	600/=	400/=
Amboni	300/=	200/=

(a) How should the delivery be scheduled in order that the transportation cost is minimum?

(b) After meeting the orders, how many kilograms of rice will remain at Donge godown?

7. (a) Prove that $\cos(2 \tan^{-1} x) = \frac{1-x^2}{1+x^2}$

(b) Find the expression involving θ for an approximation of $\frac{\sin \theta + 1}{5 + 3 \tan \theta - 4 \cos \theta}$ for small value of θ

(c) Show that $\operatorname{cosec} 2\theta + \cot 2\theta = \cot \theta$, hence solve the equation $\operatorname{cosec} 2\theta + \cot 2\theta = 2$ for $0^\circ \leq \theta \leq 360^\circ$

8. (a) Solve the inequality $\frac{x^2 - 5x + 6}{x^2 - 4} \geq 1$

(b) Find the coefficient of x^3 in the expansion $\frac{3+x}{1+3x}$

(c) Find the value of n if $\sum_{k=1}^n 2^{2k-1} = 31$

9. (a) Find (i) $\int \frac{dx}{x(x^2 + 5)}$

(ii) $\int \sec^3 x \, dx$

(b) The gradient of a curve at point (x, y) is $1 + \frac{1}{2}x^2$ and the curve passes through the point $x=1$ and $y=0$. Find the equation of the curve and the area bounded by it, the x -axis, $x=1$ and $x=3$

10. (a) Given $f(x, y) = xe^{xy}$, find

i. $\frac{\partial(1,3)}{\partial x}$

ii. $\frac{\partial(2,3)}{\partial y}$

(b) Find $\frac{dy}{dx}$ if $xy - xe^{-y} + \tan x = \ln x^2$

(c) Use Taylor's theorem to obtain a series expansion for $\cos\left(x + \frac{\pi}{3}\right)$ starting terms including that in x^3 .