

THE UNITED REPUBLIC OF TANZANIA
REGIONAL ADMINISTRATION AND LOCAL GOVERNMENT
FORM SIX JOINT EXAMINATION,
ADVANCED MATHEMATICS 2

Code: 142/2

Time: 3 Hours

Tuesday, 03rd November ,2020

Instructions

1. This paper consists of sections A and B with a total of **eight (8)** Questions.
2. Answer **all** questions in section A and **two (2)** questions from section B.
3. Sections A carries **sixty (60)** and section B carries **forty (40)** marks.
4. **All** necessary workings and answers for each question done must be shown clearly.
5. Mathematical tables and non-programmable calculators may be used.
6. Cellular phones and any unauthorized materials are **not** allowed in the examination room.
7. Write your **Examination Number** on every page of your answer sheet(s).

SECTION A (60 Marks)

Answer **all** questions in this section.

1. a) i. Given that the events A and B are such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = Y_1$ if they are mutually exclusive and $P(B) = Y_2$ if they are independent. Find the value of $Y_1 + Y_2$
 - ii. A box contains nine correction pen, numbered from one to nine. A correction pen is Drawn the box. If the number even, then a coin is tossed. If the number is prime, then a die is thrown. How many outcomes are possible?
 - b) A family of 6 people seat on a round table for dinner. In how many ways can it be done? if
 - i. father and mother next to each other
 - ii. father and mother not next to each other
 - c) Given that the probability that an individual is suffering from Malaria is 0.001. Determine the probability that out of 2000 individuals at least 2 will suffer
2. (a) Write the contra positive and converse of the following logical statements
 - i. "you cannot comprehend geometry if you do not know how to reason deductively"
 - ii. If two integers a and b are such that a is greater than b, then a-b is always a positive integer
 - iii. If a quadrilateral is rectangle, then it has two pairs of parallel sides
 - (b) (i) Use a truth table to determine whether the following statement is valid or not
 "If I work hard, then I will get praise. If I get praise then I will not have to get a second job. I got a second job. Therefore, I did not work hard"
 - (ii) Use laws of Algebra to simplify the $[(p \wedge q) \vee [(-r \wedge (q \wedge p))]]$ and then use the results to draw an electrical network corresponding to the proposition.
 - (c) A statement J is formed from P, G and M write J in most simplified form and use It to draw an electric circuit which will allow the current to flow whenever J is true
3. (a) (i) A force of magnitude 10 moves an object from A(1,2,3) to point B(3,4,5). If the force acts on the direction of the vector $a = 2i - 2j + k$, Find the work done by force
 - (b) If $\Phi(x, y, z) = x^2 yz$ and $a = 3x^2 yi + yzj - xzk$ find $\frac{\partial^2}{\partial y \partial z}(\Phi \cdot a)$ at the point (1, -2, -1)
 - (c) If θ is the angle between two point unit vectors a and b. show that $\frac{1}{2}(a + b) = \cos\left(\frac{\theta}{2}\right)$

- (d) a particle travels so that its acceleration is given by $a = 2e^{-t}i + 5\cos t j - 3\sin t k$. if the particle is located at $(1, -3, 20)$ at time $t=0$ and is moving with a velocity given by $4i - 3j + 2k$
4. (a). Show that the roots of the equation $x^2 - 2x\cos\theta + 1 = 0$ are $\cos\theta + i\sin\theta$ and $\cos\theta - i\sin\theta$
- (b) Given the complex number $Z = \sin\left(\frac{\pi}{5}\right) + i\left(1 - \cos\left(\frac{\pi}{5}\right)\right)$, Find its modulus and argument.
- Express your answer with π
- (c) If $\frac{(3+4i)(1+2i)}{1+3i} = q(1+i)$, $q \in \mathbb{R}$. Find the value of q . Hence simplify $\arctan\left(\frac{4}{3}\right) + \arctan 2 - \arctan 3$ And leave your answer in π
- (d) If imaginary part of $\frac{2z+1}{iz+1}$ is -2 , where Z is a complex number then show that the locus of the point representing z is a straight line.

SECTION B

Answer **Two (2)** question from this section.

5. (a) If $\tan\alpha = \frac{m}{m+1}$, and $\tan\beta = \frac{1}{2m+1}$, then find the value of $\alpha + \beta$
- (b) Given $24\cos\left(\frac{\pi}{3}x\right) + 10\sin\left(\frac{\pi}{3}x\right) = 13$, without using t-formula, show that $x = (6n \pm 1) + \frac{3}{n}\tan^{-1}\left(\frac{5}{12}\right)$ where n is an integer
- (c) (i) Find the maximum and minimum value of $\frac{1}{2\cos\theta + 5\sin\theta + 7}$
- (ii) Prove that $\frac{\tan x + \sec x - 1}{\tan x - \sec x + 1} = \frac{1 + \sin x}{\cos x}$
6. (a)(i) If $a^x = \left(\frac{a}{k}\right)^y = k^m$, where $a \neq 1$, show that $y = \frac{mx}{m-x}$
- (ii) Express $\frac{8x^2 - x - 1}{(x-1)^2}$ in partial fraction
- (b) (i) Find the sum of the first n terms of the series $2 \times 3 + 3 \times 4 + 4 \times 5 + \dots$
- (ii) Find the sum of the first n terms of the series $\frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots$ and hence evaluate sum to infinity of the series
- (c) (i) By using mathematical induction, prove that $2^{n+2} + 3^{2n+1}$ is divisible by 7 for all positive integers

(ii) Solve $\frac{2x^2 - 7x - 4}{3x^2 - 14x + 11} \leq 2$

7. (a) Form the Differential equation from $y = (A \cos x + B \sin x)e^{-x}$

(b)(i) Solve $\frac{dy}{dx} = \left(\frac{2x + y - 1}{x - 2} \right)^2$

(ii) Solve the Differential equation $(x + 1) \frac{dy}{dx} - y = e^x (1 + x)^2$

(c) (i) Given that $(1 + x^2) \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} = 0$, show that $y = \tan^{-1} x + c$

(d) Suppose the initial temperature of the body was θ_1 and after time 't' the temperature of the body is θ_2 . Prove that $T = A + B e^{-kt}$, where T is the temperature of the body after time 't', A is the temperature of the surrounding (constant) and B is the temperature difference between the body and the surrounding

8. (a) Define the following

- i. Directrix
- ii. Latus Rectum
- iii. Focal chord
- iv. Eccentricity

(b) Find the coordinates of the vertex, focus and the equation of directrix of each of the following (i) $(y + \sqrt{x})(y - \sqrt{x}) = 0$ (ii) $x^2 - 4y + 12 = 0$

(c) Find the equation of the parabola whose focus is (3,0) and directrix is $3x + 4y = 1$

(d) Change the Cartesian equation $\frac{1}{4}(x^2 + y^2 - 4y)^2 = x^2 + y^2$ into polar equation and then sketch the corresponding curve of the equation above.