TANZANIA HEAD OF ISLAMIC SCHOOL COUNCIL

FORM SIX INTER ISLAMIC MOCK EXAMINATION

142/2

ADVANCED MATHEMATICS 2

MARKING GUIDE

1 (a) $^4C_2 = \frac{4!}{(4-2)!\,2!} = \frac{4!}{2!\,2!} = \frac{4\times3\times2\times1}{2\times2}$ 02
$= 6$ 01

b) $E(x) = 1$

$\frac{1}{6} + \frac{1}{3} + \frac{1}{4} + x + y = 1$ 01

$x + y = \frac{1}{4}$ --- (i) 01

$\sum x\,P(x) = \frac{14}{3}$ 02

$2\left(\frac{1}{6}\right) + 3\left(\frac{1}{3}\right) + 5\left(\frac{1}{4}\right) + 7x + 9y = \frac{14}{3}$

$7x + 9y = \frac{25}{12}$ --- (ii) 01

(i) + (ii)

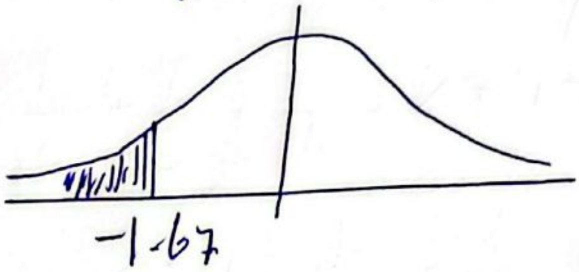
$x = \frac{1}{12}$, $y = \frac{1}{6}$ 01

1/20

(e) (i) P(less than 750)

$P(X < 750) = P\left(Z < \frac{750-753}{18}\right)$ ✓1

$P(Z < -1.67)$



✓1

$P(Z < -1.67) = 0.0475$ ✓1

For the batch of 1500 bottles, it is likely that

$1500 \times 0.0475 = 71$ bottles. ✓1

will contain less than 750 ml.

(ii) $Z_1 = \frac{751-753}{18}$ and $Z_2 = \frac{754-753}{1.8}$ ✓1

-1.11 and 0.556. ✓½

$P(751 < X < 754) = P(-1.11 < Z < 0.556)$

$0.3663 + 0.6123$ ✓1

0.5788

7/28

For 1500 bottles it is likely that
$1500 \times 0.5788 = 868$ bottles ✓1

2 (a) (i) $(p \wedge q) \vee r$ –
Either both today is Friday and Asha is clever or you will come tomorrow. ✓1

(ii) $(p \vee q) \wedge r$
Both either today is Friday or Asha is clever and you will come tomorrow ✓1

(iii) $(p \vee \sim r) \to \sim p$.
If either today is Friday or you won't come tomorrow then ~~Asha is not~~ You won't come tomorrow. ✓1

(iv) $(q \to p) \vee (p \to \sim r)$
Either if Asha is clever then today is Friday or if today is Friday then you will not come tomorrow. ✓1

3/28

b) p is T ½

q is T ½

r is F ½

(i) $(p \wedge q) \vee r$ ½

$(T \wedge T) \vee F$ ½

$T \vee F$

True ½

(ii) $(p \vee r) \wedge r$ ½

$(T \vee F) \wedge F$ ½

$T \wedge F$

False ½

(iii) $(q \rightarrow p) \vee (p \rightarrow \neg r)$

$(T \rightarrow T) \vee (T \rightarrow \neg F)$ ½

$T \vee (T \rightarrow T)$ ½

$T \vee T$ ½

True

4/28

2 (c) Let p = He borrow mathematics' book

q = He will remain in school $\sqrt{1}\frac{1}{2}$

r = He is punished

$[(p \to q) \wedge (r \vee \sim q) \wedge \sim r] \to \sim p$

p	q	r	~p	~q	~r	p→q (a)	r∨~q (b)	a∧b∧~r (c)	c → ~p
T	T	T	F	F	F	T	T	T	F
T	T	F	F	F	T	T	F	F	T
T	F	T	F	T	F	F	T	F	T
T	F	F	F	T	T	F	T	F	T
F	T	T	T	F	F	T	T	F	T
F	T	F	T	F	T	T	F	F	T
F	F	T	T	T	F	T	T	F	T
F	F	F	T	T	T	T $\sqrt{\frac{1}{2}}$	T	T $\sqrt{\frac{1}{2}}$	T $\sqrt{\frac{1}{2}}$

It is not tautology $\sqrt{\frac{1}{2}}$

3 (a) $\cos 2A + \cos 2B + \cos 2C$ --- (i)

$\cos 2A = 2\cos^2 A - 1$ --- (ii) $\sqrt{1}$

$1 + \cos 2B = 2\cos^2 B$ ---

$\cos 2B = 2\cos^2 B - 1$ --- (iii) $\sqrt{1}$

Add $1 + \cos 2C = 2\cos^2 C$

$\cos 2C = 2\cos^2 C - 1$ --- (iv) $\sqrt{1}$

5/08

Subst- (ii), (iii), (iv) into (i)

$\cos 2A + \cos 2B + \cos 2C$ ✓½

$2\cos^2 A - 1 + (2\cos^2 B - 1) + (2\cos^2 C - 1)$

$2\cos^2 A + 2\cos^2 B + 2\cos^2 C - 3$

$2\underbrace{(\cos^2 A + \cos^2 B + \cos^2 C)}_{1} - 3$ ✓1

Sum of the squares of direction cosines = 1

$2(1) - 3$

$= -1$ ✓½

$\cos 2A + \cos 2B + \cos 2C = -1$

$R.F = F_1 + F_2 + F_3$

$F_R = \begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 \\ 6 \\ -9 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$ ✓1

$F = \begin{pmatrix} 7 \\ 5 \\ -10 \end{pmatrix}$ ✓½

6/kg

$$\underset{\sim}{d} = 7 \times \left(\frac{-2i + 10j + 11k}{\sqrt{2^2 + 10^2 + 11^2}} \right) \quad 1$$

$$= \frac{7}{15}\left(-2\underset{\sim}{i} + 10\underset{\sim}{j} + 11\underset{\sim}{k}\right) \quad \frac{1}{2}$$

W-done $= \underset{\sim}{F} \cdot \underset{\sim}{d}$.

$$= \frac{7}{15}\begin{pmatrix} 7 \\ 5 \\ -10 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 10 \\ 11 \end{pmatrix} \quad \frac{1}{2}$$

$$= \frac{7}{15}\left(-14 + 50 - 110\right)$$

$$= \frac{7}{15}\left(-74\right)$$

$$= \left| \frac{-518}{15} \right| \quad \frac{1}{2}$$

W-d $= \frac{518}{15}$ units

$= 34.533$ joules

$\frac{1}{2}$

$6/28$

3 c) $\underline{v} = 6t^2\underline{i} + 4e^{-2t}\underline{j} + 8\cos 4t\,\underline{k}$ $0\frac{1}{2}$

from

$$v = \frac{dr}{dt}$$

$$r = \int \underline{v}\,dt$$

$$r(t) = \int \left(6t^2\underline{i} + 4e^{-2t}\underline{j} + 8\cos 4t\,\underline{k}\right) dt$$

$$r(t) = \left[2t^3\hat{i} - 2e^{-2t}\hat{j} + 2\sin 2t\,\underline{k}\right] + \underline{c}$$ 01

at $r(0) = \hat{i} + \hat{j} + \underline{k}$

$$\hat{i} + \hat{j} + \underline{k} = 2(0)\hat{i} - 2e^{-2(0)}\hat{j} + 2\sin 2(0)\,\underline{k} + \underline{c}$$ 01

$$\hat{i} + \hat{j} + \underline{k} = -2\hat{j} + \underline{c}$$

$$\underline{c} = \hat{i} + 3\hat{j} + \underline{k}$$ 01

from

$$r(t) = 2t^3\hat{i} - 2e^{-2t}\hat{j} + 2\sin 2t\,\underline{k} + \underline{c}$$

$$r(t) = 2t^3\hat{i} - 2e^{-2t}\hat{j} + 2\sin 2t\,\underline{k} + \hat{i} + 3\hat{j} + \underline{k}$$ 01

$$\therefore r(t) = (1 + 2t^3)\hat{i} + (3 - 2e^{-2t})\hat{j} + (1 + 2\sin 2t)\underline{k}$$

01

7/28

4 (a)

$$\frac{x-2+(y-3)i}{1+i} = 1-3i$$

$$\frac{(x-2)+(y-3)i)(1-i)}{1+i} = 1-3i \quad \checkmark\tfrac{1}{2}$$

$$\frac{x-2-i(x-2)+i(y-3)-i^2(y-3)}{1+1} = 1-3i \quad \checkmark\tfrac{1}{2}$$

$(Z\bar{Z} = x^2+y^2)$

$$\frac{(x-2+y-3)+i(y-3-x+2)}{} \quad \checkmark\tfrac{1}{2}$$

Comparing

$x+y-5=2$

$x+y=7$ ---- (i) $\checkmark\tfrac{1}{2}$

$-x+y-1=-6$

$-x+y=-5$ ---- (ii) $\checkmark\tfrac{1}{2}$

solve (i) and (ii)

$x=6,\ y=1$

4 (b) $P(x) = x^4-4x^3+12x^2+4x-13$

$x=2-3i$ when $x=2+3i$ is also a root $\checkmark\tfrac{1}{2}$ $\checkmark 1$

The equation formed by $2-3i$ and $2+3i$

8/28

$$\begin{array}{r|l} & x^2-1 \\ \hline x^2-4x+13 & x^4-4x^3+12x^2+4x-13 \\ & \underline{x^4-4x^3+13x^2} \\ & \quad -x^3+4x-13 \\ & \quad \underline{-x^3+4x-13} \\ & \quad --- \end{array}$$

√2

$x^2-1=0$

$x=\pm 1$

√

the other roots are $2+3i$, 1, and -1 √½

9/20

4 c)

$$|z+1| = 2|z-1+3i|$$

let $z = x+iy$. $\frac{1}{2}$

$$|x+iy+1| = 2|x+iy-1+3i| \quad \frac{1}{2}$$

$$|x+1+iy| = 2|x-1+(y+3)i|. \quad \frac{1}{2}$$

$$\sqrt{(x+1)^2+y^2} = 2\sqrt{(x-1)^2+(y+3)^2} \quad \frac{1}{2}$$

Squaring both sides.

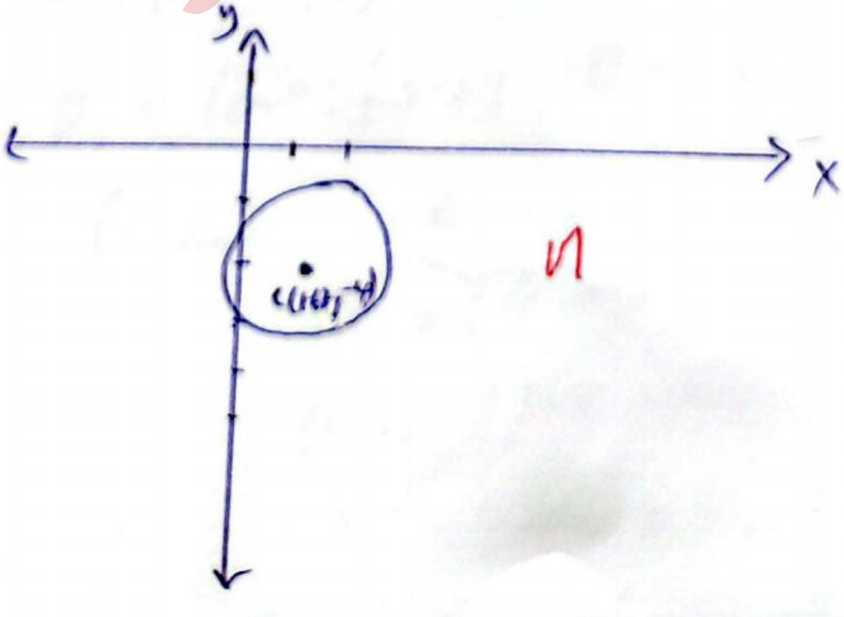
$$(x+1)^2+y^2 = 4\left((x-1)^2+(y+3)^2\right). \quad \frac{1}{2}$$

$$x^2+2x+1+y^2 = 4(x^2-2x+1+y^2+6y+9). \quad 1$$

$$x^2+2x+1+y^2 = 4x^2-8x+4+4y^2+30y+36 \quad 1$$

$$3x^2+3y^2-10x+24y+39 = 0.$$

centre $= (5/3, -4)$ 1



1

5 (a) $10\sec^2\theta - 3 = 17\tan\theta$

$10(1+\tan^2\theta) - 17\tan\theta - 3 = 0$

$10\tan^2\theta - 17\tan\theta + 7 = 0$ 01

$\tan\theta = \dfrac{17 \pm \sqrt{(-17)^2 - 4\times10\times7}}{2\times10}$

$\tan\theta = \dfrac{17 \pm \sqrt{9}}{20}$

$\tan\theta = \dfrac{17 \pm 3}{20}$ 01

$\tan\theta = \dfrac{20}{20} = 1$

$\tan\theta = 7/10$

$\theta = \tan^{-1}(1)$ $\theta = 45°$

$\theta = \tan^{-1}(0.7)$ $\theta = 34.9$ 01

General soln.

$\theta = 180n + \alpha$

$\theta = 180n + 45°$ 01

$\theta = 180n + 34.9$ 01

10/18

5 (b) $\frac{\tan x - \tan y}{\cot x - \cot y} = \tan x \tan y$

L.H.S

$\frac{\tan x - \tan y}{\cot x - \cot y} = \frac{\tan x - \tan y}{\frac{1}{\tan x} - \frac{1}{\tan y}}$ 01

$\frac{\tan x - \tan y}{\frac{\tan y - \tan x}{\tan x \tan y}} = \frac{\tan x \tan y}{(\tan y - \tan x)} \times (\tan x - \tan y)$ 01

$= \tan x \tan y$

5 (c) $h = 4\cos 2x + 3\sin 2x$

$= R\sin(2x + \alpha)$ 01

$= R\sin 2x \cos\alpha + R\sin\alpha\cos 2x$ 01

$R\cos\alpha = 3$

$R\sin\alpha = 4$ 01

$\tan\alpha = \frac{4}{3}, \ \alpha = \tan^{-1}\left(\frac{4}{3}\right)$ 01

$\alpha = 53.1^\circ$

$R = \sqrt{4^2+3^2} = \sqrt{25} = 5$ 01

For maximum

$R\sin(2x+\alpha) = 1$ 01

$h = 5$ units

Maximum height $= 5$ units 01

5 (d) i) $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x$

L.H.S

$\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ $\cancel{x} = \tan\theta$ 01

$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}\left(\frac{1-\tan\theta}{1+\tan^2\theta}\right)$ 01

$\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta$

$\sin^{-1}(\sin 2\theta)$ 01 $= 2\tan^{-1}x$

$= 2\theta$

$= 2\tan^{-1}x$ # 01

17/28

5 (d) (ii)

$\sin^{-1}x = 2\tan^{-1}y$

let $A = \tan^{-1}y$.

$\cancel{\tan A =} \Rightarrow \tan A = y$. 01

from

$\sin^{-1}x = 2\tan^{-1}y$

$\sin^{-1}x = 2A$.

$x = \sin 2A$. 01

but $\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$.

so

$x = \dfrac{2\tan A}{1+\tan^2 A}$ 1/2

$x = \dfrac{2y}{1+y^2}$. 1/2

proved.

13/13

6 (e) $$\frac{x^3-2x^2-4x-4}{x^2+x-2}$$

$$\begin{array}{r} x-3 \\ x^2+x-2 \overline{\big)\, x^3-x^2-4x-4} \\ x^3+x^2-2x \\ \hline -3x^2+2x-4 \\ -3x^2-3x+6 \\ \hline x-10 \end{array}$$ 01

$$\frac{x^3-2x^2-4x-4}{x^2+x-2} = x+3+\frac{x-10}{x^2+x-2}$$ 01

$$\frac{x-10}{(x+2)(x-1)} = \frac{A}{x+2}+\frac{B}{x-1}$$ 1

$$= \frac{A(x-1)+B(x+2)}{(x+2)(x-1)}$$ ½

$$x-10 = A(x-1)+B(x+2)$$ ½

14/28

$$\frac{x-10}{(x+2)(x-1)} = \frac{4}{x+2} - \frac{3}{x-1} \quad \tfrac{1}{2}$$

$$\Rightarrow \underline{x-3} + \frac{4}{x+2} - \frac{3}{x-1} \quad \tfrac{1}{2}$$

15/28

6 (b)
$$x+y+z=37$$
$$2x-y+z=-36$$
$$x+2y-z=11$$

$$\begin{pmatrix}1 & 1 & 1\\ 2 & -1 & 1\\ 1 & 2 & -1\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}37\\ 36\\ -11\end{pmatrix}$$ √1

$$=\begin{vmatrix}-1 & 1\\ 2 & -1\end{vmatrix}-\begin{vmatrix}2 & 1\\ 1 & -1\end{vmatrix}+\begin{vmatrix}2 & -1\\ 1 & 2\end{vmatrix}$$

$$=(1-2)-(-2-1)+(4+1)$$ √1

$$=7$$

$$x=\frac{\begin{vmatrix}37 & 1 & 1\\ 36 & -1 & 1\\ -11 & 2 & -1\end{vmatrix}}{7}=\frac{49}{7}=7$$ √½

$$y=\frac{\begin{vmatrix}1 & 37 & 1\\ 2 & 36 & 1\\ 1 & -11 & -1\end{vmatrix}}{7}=\frac{28}{7}=4$$ √½

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$$z = \frac{\begin{vmatrix} 1 & 1 & 37 \\ 2 & -1 & 36 \\ 1 & 2 & -11 \end{vmatrix}}{7} = \frac{182}{7} = 26$$ ✓½

The digit passcode is 7426 ✓½

6 (c) $$\frac{1}{(4-x)^2} = \frac{1}{(4(1-\frac{x}{4}))^2} = \frac{1}{4^2(1-\frac{x}{4})^2}$$

$$\frac{1}{16(1-\frac{x}{4})^2} = \frac{1}{16(1-\frac{x}{4})^2}$$ (0)

$$= \frac{1}{16}\left(1-\frac{x}{4}\right)^{-2}$$ ✓½

$$\frac{1}{16}\left[1+(-2)\left(-\frac{x}{4}\right)+\frac{(-2)(-3)}{2!}\left(-\frac{x}{4}\right)^2+\frac{(-2)(-3)(-4)}{3!}\left(-\frac{x}{4}\right)^3+\cdots\right]$$ ✓½

$$= \frac{1}{16}\left(1+\frac{x}{2}+\frac{3x^2}{16}+\frac{x^3}{16}+\cdots\right)$$ ✓½

17/18

It is valid for $\left|\frac{x}{4}\right| < 1$ ✓½

that is $-4 < x < 4$ ✓½

(c) $\sum_{x=1}^{n} 2k(k-4)(k+6) = \sum_{k=1}^{n} 2k^3 + 4k^2 - 48k$ ✓1

$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$, $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$ ✓½

$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$ ✓½

$= \frac{n(n+1)}{6}\left(3n(n+1) + 4(2n+1) - 24\right)$ ✓½

$= \frac{n(n+1)}{6}\left(3n(n+1) + 4(2n+1) - 144\right)$ ✓1

$= \frac{n(n+1)}{6}\left(3n^2 + 11n - 140\right)$ shown ✓½

7 a) Given

$$(x-a)^2 + y^2 = a^2 \quad \text{---(i)}$$

$$2(x-a) + 2y\frac{dy}{dx} = 0 \quad \tfrac{1}{2}$$

$$x - a + y\frac{dy}{dx} = 0 \quad \tfrac{1}{2}$$

$$a = x + y\frac{dy}{dx} \quad \text{---(ii)} \quad \tfrac{1}{2}$$

Substitute eqn (ii) into (i)

$$\left(x - \left(x + y\frac{dy}{dx}\right)\right)^2 + y^2 = \left(x + y\frac{dy}{dx}\right)^2 \quad \tfrac{1}{2}$$

$$\left(-y\frac{dy}{dx}\right)^2 + y^2 = x^2 + 2xy\frac{dy}{dx} + y^2\left(\frac{dy}{dx}\right)^2 \quad \tfrac{1}{2}$$

$$y^2\left(\frac{dy}{dx}\right)^2 + y^2 = x^2 + 2xy\frac{dy}{dx} + y^2\left(\frac{dy}{dx}\right)^2 \quad \tfrac{1}{2}$$

$$y^2 = x^2 + 2xy\frac{dy}{dx}$$

$$2xy\frac{dy}{dx} = y^2 - x^2 \quad \tfrac{1}{2}$$

$$\therefore \frac{dy}{dx} = \frac{y^2 - x^2}{2xy} \quad \tfrac{1}{2}$$

$1\ \tfrac{2}{18}$

7 b) $\ln(y+1) + \left(\frac{x}{y+1}\right)\frac{dy}{dx} = \frac{1}{x(x+1)}$.

This is exact.

$$\frac{d}{dx}\left(x\ln(y+1)\right) = \frac{1}{x(x+1)} \quad \tfrac{1}{2}$$

$$d\left(x\ln(y+1)\right) = \frac{1}{x(x+1)}\,dx. \quad \tfrac{1}{2}$$

$$\int d(x\ln(y+1)) = \int \frac{1}{x(x+1)}\,dx$$

$$x\ln(y+1) = \int\left(\frac{A}{x} + \frac{B}{x+1}\right)dx \quad \tfrac{1}{2}$$

take

$$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1} = \frac{A(x+1) + Bx}{x(x+1)}. \quad \tfrac{1}{2}$$

$$A = 1, \quad B = -1. \quad 1$$

$$\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1} \quad \tfrac{1}{2}$$

$$x\ln(y+1) = \int\left(\frac{1}{x} - \frac{1}{x+1}\right)dx \quad \tfrac{1}{2}$$

$$x\ln(y+1) = \ln x - \ln(x+1) + c \quad \tfrac{1}{2}$$

$$\ln(y+1)^x = \ln\left(\frac{x}{x+1}\right) + \ln A \quad \tfrac{1}{2}$$

$$\ln(y+1)^x = \ln\left(\frac{Ax}{x+1}\right)$$

$$(y+1)^x = \frac{Ax}{x+1}$$ ✓1

7 b) Given

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{2x}.$$

Consider complementary part

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$$ ✓½

it's A.Q.E will be

$$m^2 - 3m + 2 = 0$$ ✓½

$$m = 2, \; m = 1$$

so

$$y_c = Ae^x + Be^{2x}.$$

For particular integral part

Let

$$y = kxe^{2x}$$ ✓½

$$y' = 2kxe^{2x} + ke^{2x}.$$

$$y'' = 4kxe^{2x} + 4ke^{2x}.$$ ✓½

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7 c) From

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{2x}. \quad \checkmark_{\frac{1}{2}}$$

$$(4kxe^{2x} + 4ke^{2x}) - 3(2kx + ke^{2x}) + 2kxe^{2x} = e^{2x}.$$

$$ke^{2x} = e^{2x}. \quad \checkmark_{\frac{1}{2}}$$

So $k = 1$

$$y_p = xe^{2x} \quad \checkmark_{\frac{1}{2}}$$

from

$$y = y_c + y_p.$$

$$y = Ae^{x} + Be^{2x} + xe^{2x}.$$

$\therefore$ The solution is $y = Ae^{x} + Be^{2x} + xe^{2x}$. $\checkmark 1$

d) Let $\phi(t)$ be the amount of salt in tank at ~~time t~~ time t (min).

Volume of solution

$$V(t) = 10 + (3-2)t.$$

$$= 10 - t. \quad \checkmark_{\frac{1}{2}}$$

$$\frac{d\phi}{dt} = \text{rate in} - \text{rate out}$$

$\checkmark_{\frac{1}{2}}$

$\frac{7\frac{1}{2}}{20}$

7 a)

$$\text{Conc} = \frac{Q}{10+t}$$

then

$$-\frac{dQ}{dt} = \frac{2Q}{10+t} \quad \frac{1}{2}$$

By separating variable.

$$\frac{dQ}{Q} = 2$$

$$\int_{20}^{Q} -\frac{dQ}{Q} = \int_{0}^{5} \frac{2\,dt}{10+t}. \quad \frac{1}{2}$$

$$\ln Q\Big|_{20}^{Q} = -2\Big[\ln(10+t)\Big]_0^5 \quad \frac{1}{2}$$

$$\ln\left(\frac{Q}{20}\right) = -2(\ln 15 - \ln 10). \quad \frac{1}{2}$$

$$\ln\frac{Q}{20} = -2\ln(15/10). \quad \frac{1}{2}$$

$$\ln(Q/20) = \ln\left(\frac{15}{10}\right)^{-2} \quad \frac{1}{2}$$

$$\frac{Q}{20} = \left(\frac{15}{10}\right)^{-2} \Rightarrow Q = \frac{20 \times 100}{150} \quad \frac{1}{2}$$

$$Q = 8.889\,\text{kg}. \quad \frac{1}{2}$$

∴ After 5 minutes the tank will contain 8.89 kg of salt. $\frac{1}{2}$

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8 a) Given

$$y^2 = 8x$$

Slope of tangent, $m = 1$

From

$$y = mx + c$$

$$y = x + c \quad \checkmark\tfrac{1}{2}$$

Substituting eqn of tangent into the equation of parabola.

$$(x+c)^2 = 8x. \quad \checkmark\tfrac{1}{2}$$

$$x^2+2xc+c^2 = 8x. \quad \checkmark\tfrac{1}{2}$$

$$x^2+2xc+c^2-8x = 0 \quad \checkmark\tfrac{1}{2}$$

$$x^2+(2c-8)x+c^2=0 \quad \checkmark\tfrac{1}{2}$$

For tangent line,

$$b^2 = 4ac$$

$$(2c-8)^2 = 4(1)c^2 \quad \checkmark\tfrac{1}{2}$$

$$4c^2-32c+64 = 4c^2 \quad \checkmark\tfrac{1}{2}$$

$$-32c = -64. \quad \checkmark\tfrac{1}{2}$$

$$c = +2$$

From

$$y = x + c \quad \checkmark\tfrac{1}{2}$$

$$y = x + 2 \quad \checkmark\tfrac{1}{2}$$

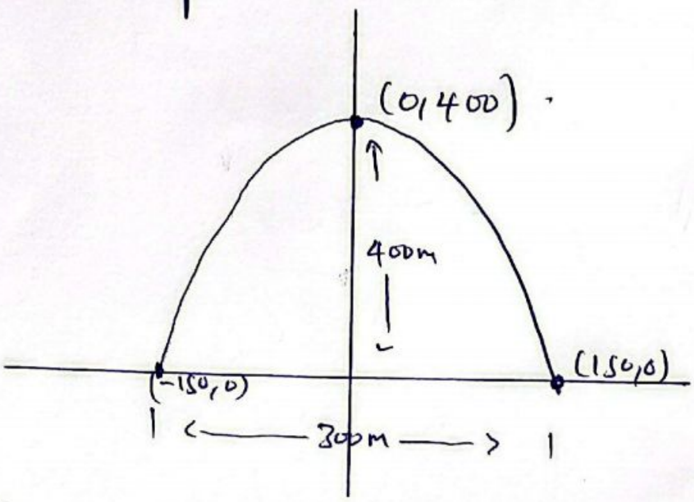
$\therefore$ The eqn of tangent will be $y = x - 2$

5/8

8 b) Given, height = 400m

distance = 300m.

This is parabola that opens downward



from

$$(x-h)^2 = 4b(y-k).$$

~~h = 0 , k = 400 since~~

$$(h,k) = (0,400) \Rightarrow h=0, \; k=400$$

$$x^2 = 4b(y-400).$$

at (150, 0)

$$150^2 = 4b(0-400).$$

$$150^2 = -1600b.$$

$$b = -14.0625$$

the equation will be

$$x^2 = -4(-14.0625)(y-400)$$

$$x^2 = -56.25(y-400).$$

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8 c) Given

$$x^2-8x+4y^2+12=0$$

$$x^2-8x+16+4y^2+12=16$$ $\checkmark\frac{1}{2}$

$$(x-4)^2+4y^2=4$$

$$\frac{(x-4)^2}{4}+\frac{y^2}{1}=1$$ $\checkmark\frac{1}{2}$

$a^2=4$

$b^2=1$ $\checkmark\frac{1}{2}$

$a=4$ and $b=1$ $\checkmark$

Since $a>b$

$$b^2=a^2(1-e^2)$$ $\checkmark\frac{1}{2}$

$$1=4(1-e^2)$$

$$e^2=1-\frac{1}{4}$$ $\checkmark\frac{1}{2}$

$$e^2=\frac{3}{4}$$

$$e=\frac{\sqrt{3}}{2}$$ $\checkmark\frac{1}{2}$

i) Foci are $(\pm ae+h, 0)$

$(h,k)=(4,0)$ $\checkmark\frac{1}{2}$

Foci $=\left(\pm\frac{\sqrt{3}}{2}+4, 0\right)=(\pm\sqrt{3}+4, 0)$ $\checkmark\frac{1}{2}$

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∴ Foci are $(\sqrt{3}+4, 0)$ and $(-\sqrt{3}+4, 0)$. ✓½

ii) Vertices = $(\pm a, 0)$.

$= (\pm 2, 0)$ ✓½

∴ Vertices are $(2,0)$ and $(-2,0)$. ✓

iii) Equation of directrices, $x = \pm \frac{a}{e} + h$ ✓

$x = \pm \frac{2}{\sqrt{3}/2} + 4$. ✓½

$x = \pm \frac{4}{\sqrt{3}} + 4 = \pm \frac{4\sqrt{3}}{3} + 4$. ✓½

∴ Directrices are $x = \frac{4\sqrt{3}}{3} + 4$ and $x = -\frac{4\sqrt{3}}{3} + 4$. ✓½

5 d) ~~Co~~ $r = 3 - 3\sin\theta$

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
r	3	1.5	0.4	0	0.4	1.5	3	4.5	5.6	6	5.6	4.5	3

✓2½

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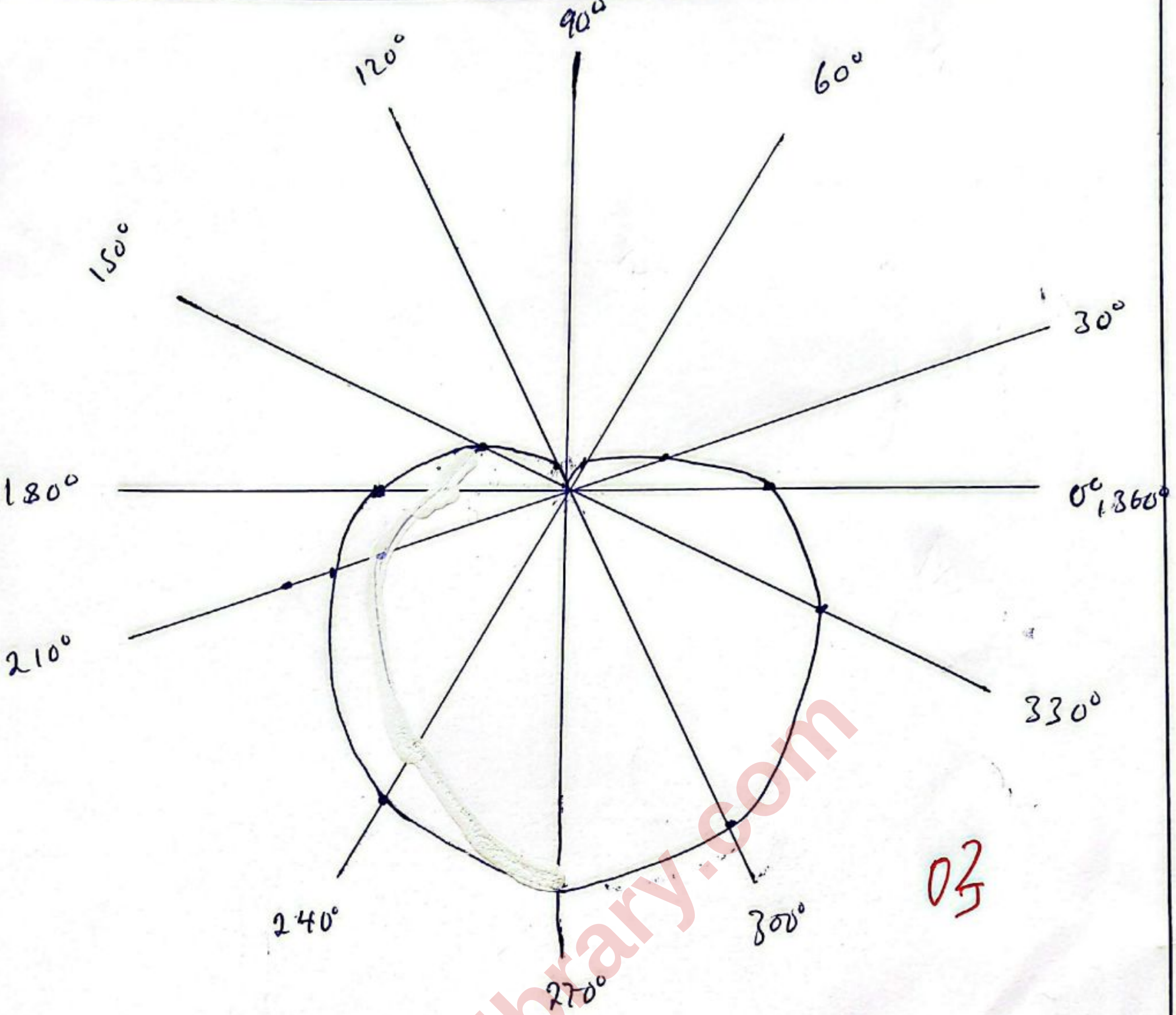

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