

TANZANIA HEADS OF ISLAMIC SCHOOL
COUNCIL

FORM SIX INTER ISLAMIC
MOCK EXAMINATION

142/1

ADVANCED MATHEMATICS I

MARKING GUIDE

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$$1) \text{ a) } \Rightarrow 3.659 \dots 03$$

$$\text{ii) } 4.877 \dots 03$$

$$\text{b) } R = 10.0677 \dots 04$$

2) a) Given that

$$\frac{1 + \tanh x}{1 - \tanh x} = e^{2x}.$$

Consider L.H.S.

$$\frac{1 + \tanh x}{1 - \tanh x} = \frac{1 + \frac{\sinh x}{\cosh x}}{1 - \frac{\sinh x}{\cosh x}} \dots 01$$

$$= \frac{\cosh x + \sinh x}{\cosh x - \sinh x} \dots \dots \dots 00\frac{1}{2}$$

$$= \frac{e^x}{e^{-x}} \dots \dots \dots 00\frac{1}{2}$$

$$= e^{2x} \dots \dots \dots 01$$

$$\therefore \frac{1 + \tanh x}{1 - \tanh x} = e^{2x}.$$

(01/30)

2
b) Given

1) $x = t + \ln(\cosh t)$, $y = \sinh t$

$$\frac{dx}{dt} = 1 + \frac{\sinh t}{\cosh t} \quad \dots \dots \dots \text{oo}^{1/2}_2$$

$$\frac{dy}{dt} = \cosh t \quad \dots \dots \dots \text{oo}^{1/2}_2$$

from

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}$$

$$\frac{dy}{dx} = \frac{\cosh t}{1 + \frac{\sinh t}{\cosh t}}$$

$$= \frac{\cosh^2 t}{\cosh t + \sinh t} \quad \dots \dots \dots \text{oo}^{1/2}_2$$

but $\cosh t + \sinh t = e^t$.

$$\frac{dy}{dx} = \frac{\cosh^2 t}{e^t} = e^{-t} \cosh^2 t$$

$$\therefore \frac{dy}{dx} = e^{-t} \cosh^2 t \quad \dots \dots \dots \text{oo}^{1/2}_2$$

Shown,

(U²/3)

2
b) Given

1) $x = t + \ln(\cosh t)$, $y = \sinh t$

$$\frac{dx}{dt} = 1 + \frac{\sinh t}{\cosh t} \quad \dots \dots \dots \text{ool/2}$$

$$\frac{dy}{dt} = \cosh t \quad \dots \dots \dots \text{ool/2}$$

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but $\cosh t + \sinh t = e^t$.

$$\frac{dy}{dx} = \frac{\cosh^2 t}{e^t} = e^{-t} \cosh^2 t$$

$$\therefore \frac{dy}{dx} = e^{-t} \cosh^2 t \quad \dots \dots \dots \text{ool/2}$$

Shown,

(U²/3)

$$2 \quad \ln \Rightarrow x = \ln \left(\sqrt{3} \left(\frac{\sqrt{3} + 1}{2} \right) \right).$$

$$= \ln \left(\frac{3+1}{2} \right)$$

$$= \ln 2. \quad \text{--- --- --- } \text{or } 1/2$$

$$y = \sinh t = \sinh (\ln \sqrt{3}).$$

$$= \frac{e^{\ln \sqrt{3}} - e^{-\ln \sqrt{3}}}{2} \quad \text{--- --- --- } \text{or } 1/2$$

$$= \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{2}$$

$$= \frac{\sqrt{3}}{3}. \quad \text{--- --- --- } \text{or } 1/2$$

from

$$\frac{d^2 y}{dx^2} = e^{-2 \ln \sqrt{3}} (\cosh^2 (\ln \sqrt{3}) (2 \sinh \ln \sqrt{3} - \cosh \ln \sqrt{3})).$$

$$= 0.$$

Hence the point $(\ln 2, \frac{\sqrt{3}}{3})$ is inflexion point. --- --- --- 01

$\therefore$ The curve C has point $(\ln 2, \frac{\sqrt{3}}{3})$ which is inflexion point.

$(\ln 2, \frac{\sqrt{3}}{3})$

$$ii) \frac{d^2y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx} \right) / \frac{dx}{dt}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt} (e^t \cosh^2 t) / \frac{\cosh t + \sinh t}{\cosh t} \dots\dots\dots 0)$$

$$= \frac{(-e^t \cosh^2 t + 2e^t \sinh t \cosh t) \cosh t}{\cosh t + \sinh t}$$

$$= \frac{e^{-t} (2 \sinh t \cosh t - \cosh^2 t) \cosh t}{e^t} \dots\dots\dots \frac{1}{2}$$

$$= e^{-2t} \cosh t (2 \sinh t - \cosh t).$$

$$\therefore \frac{d^2y}{dx^2} = e^{-2t} \cosh t (2 \sinh t - \cosh t) \dots\dots\dots \frac{1}{2}$$

Shown.

$$iii) t = \frac{1}{2} \ln 3$$

$$t = \ln \sqrt{3}.$$

$$x = \ln \sqrt{3} + \ln (\cosh \ln \sqrt{3}) \dots\dots\dots \frac{1}{2}$$

$$= \ln (\sqrt{3} \cosh (\ln \sqrt{3})).$$

$$= \ln (\sqrt{3} (\frac{e^{\ln \sqrt{3}}}{2} + \frac{e^{-\ln \sqrt{3}}}{2}))$$

$$\left(\frac{e^{\ln \sqrt{3}}}{3} \right)$$

$$ii) \frac{d^2 y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx} \right) / \frac{dx}{dt}$$

$$\frac{d^2 y}{dx^2} = \frac{d}{dt} (e^{-t} \cosh^2 t) / \frac{\cosh t + \sinh t}{\cosh t} \dots\dots\dots 0)$$

$$= \frac{(-e^{-t} \cosh^2 t + 2e^{-t} \sinh t \cosh t) \cosh t}{\cosh t + \sinh t}$$

$$= \frac{e^{-t} (2 \sinh t \cosh t - \cosh^2 t) \cosh t}{e^t} \dots\dots\dots \text{vol } 1/2$$

$$= e^{-2t} \cosh t (2 \sinh t - \cosh t)$$

$$\therefore \frac{d^2 y}{dx^2} = e^{-2t} \cosh t (2 \sinh t - \cosh t) \dots\dots\dots \text{vol } 1/2$$

Shown.

$$iii) t = \frac{1}{2} \ln 3$$

$$t = \ln \sqrt{3}$$

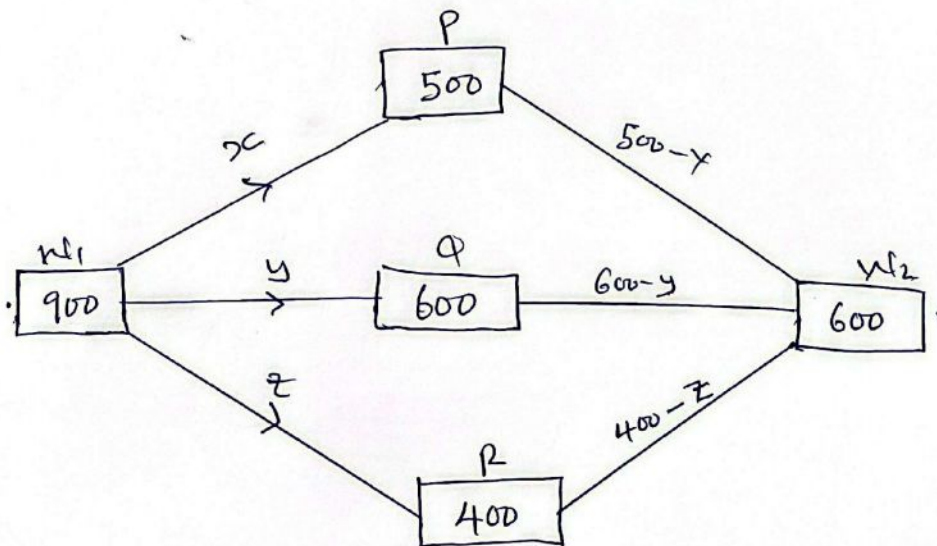
$$X = \ln \sqrt{3} + \ln (\cosh \ln \sqrt{3}) \dots\dots\dots \text{vol } 1/2$$

$$= \ln (\sqrt{3} \cosh (\ln \sqrt{3}))$$

$$= \ln (\sqrt{3} (\frac{e^{\ln \sqrt{3}} + e^{-\ln \sqrt{3}}}{2}))$$

$$\left(\frac{e^4}{3} \right)$$

a)



let

x be tonnes of bricks transported from w_1 to P . --- sol_{1/2}

y be tonnes of bricks transported from w_1 to Q . --- sol_{1/2}

then

Minimize

$$f(x, y) = 400x + 500y \quad \text{Subject}$$

Minimize

$$f(x, y) = 400x + 500y + 380000 \quad \text{--- sol}$$

Subject to

$$x \leq 500 \quad \text{--- sol}_{1/2}$$

$$y \leq 600 \quad \text{--- sol}_{1/2}$$

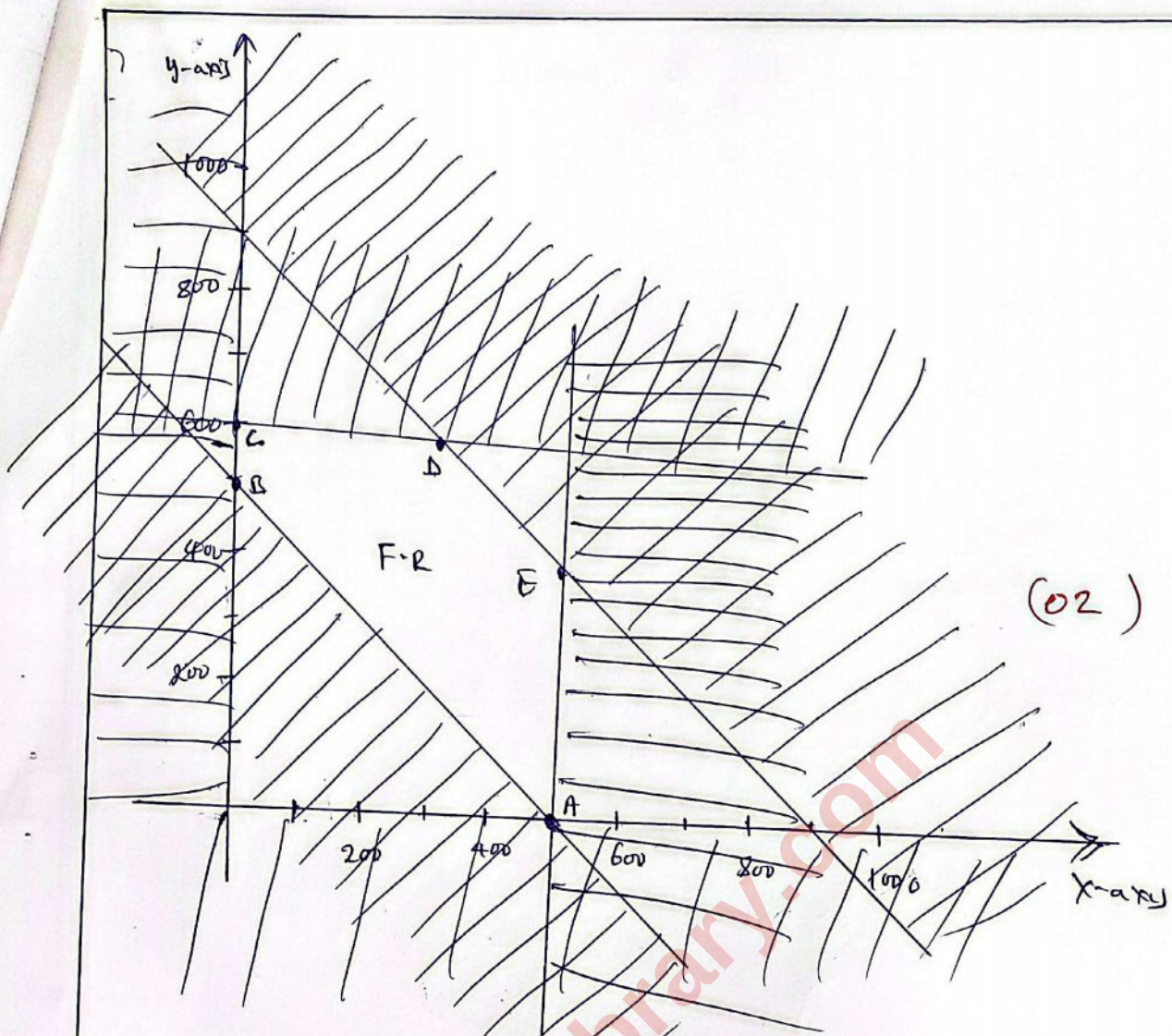
$$x + y \leq 900 \quad \text{--- sol}_{1/2}$$

$$x + y \geq 500 \quad \text{--- sol}_{1/2}$$

$$x \geq 0 \quad \text{--- sol}_{1/2}$$

$$y \geq 0 \quad \text{--- sol}_{1/2}$$

(05/31)



(02)

From the graph.

Corner points	Value of $f(x, y)$
A(500, 0)	580,000/-
B(0, 500)	630,000/-
C(0, 600)	680,000/-
D(300, 600)	800,000/-
E(500, 400)	780,000/-

-(01)

06/31

The optimal solution is at $(500, 0)$,

Therefore in order to minimize cost Hammer should transport:

500 tonnes from w_1 to P - - - - - 00 1/2

400 tonnes from w_1 to R - - - - - 00 1/2

600 tonnes from w_2 to Q - - - - - 00 1/2

b) The minimum cost of transportation is

580,000/- - - - - - 00 1/2

4. a) Given

$$x_1 = 50$$

$$\sigma^2 = 16$$

$$Y = \frac{X - 50}{4}$$

$$\bar{Y} = \frac{\sum Y}{n}$$

$$\bar{Y} = \frac{\sum \left(\frac{X - 50}{4} \right)}{n} = \frac{\frac{1}{4} \sum X - \frac{50}{4} n}{n} \dots \dots \dots$$

$$= \frac{1}{4} \frac{\sum X}{n} - \frac{50}{4}$$

$$= \frac{1}{4} \bar{X} - \frac{50}{4}$$

$$= \frac{1}{4} (50) - \frac{50}{4} = 0 \dots \dots \dots$$

$\therefore$ New mean, $\bar{Y} = 0$.

(7/31)

② For standard deviation, σ

$$\sigma_x^2 = 16,$$

$$\sigma_y^2 = ?$$

from

$$\text{Var}(ax+b) = a^2 \text{Var}(x).$$

$$\text{Var}(Y) = \text{Var}\left(\frac{x-50}{4}\right)$$

$$= \left(\frac{1}{4}\right)^2 \text{Var}(x) \dots \dots \dots \circ 1$$

$$= \left(\frac{1}{4}\right)^2 \cdot 16$$

$$= 1.$$

$$\therefore \text{Var}(Y) = 1 \dots \dots \dots \circ 1$$

"> Since all data are transformed by $Y = \frac{X-50}{4}$,
the quartiles will also transform.

$$Q_3(Y) = \frac{Q_3(X) - 50}{4} \dots \dots \dots \circ 1$$

$$= \frac{58 - 50}{4}$$

$$= 2.$$

$\therefore$ The new third quartile is 2, $\dots \dots \dots \circ 1$

8/31

b)

C.I	f	X	$X - \bar{X}$	$f(X - \bar{X})^2$	fX
20.5 - 20.9	3	20.7	-1.22	4.4652	
21.0 - 21.4	10	21.2	-0.72	5.1840	212
21.5 - 21.9	11	21.7	-0.22	0.5324	
22.0 - 22.4	13	22.2	0.28	1.0192	
22.5 - 22.9	9	22.7	0.78	5.4756	
23.0 - 23.4	2	23.2	1.28	3.2768	
	48			19.9532	1052.1

(02)

from

$$\text{Mean } \bar{X} = \frac{\sum fX}{\sum f}$$

$$= \frac{1052.1}{48} = 21.92$$

$$\bar{X} = 21.92 \quad \dots \dots \dots 01$$

$$\sigma = \sqrt{\frac{\sum f(X - \bar{X})^2}{\sum f}}$$

$$= \sqrt{\frac{19.9532}{48}}$$

$$= 0.645 \dots \dots \dots 01$$

$\therefore$ standard deviation = 0.645.

9/31

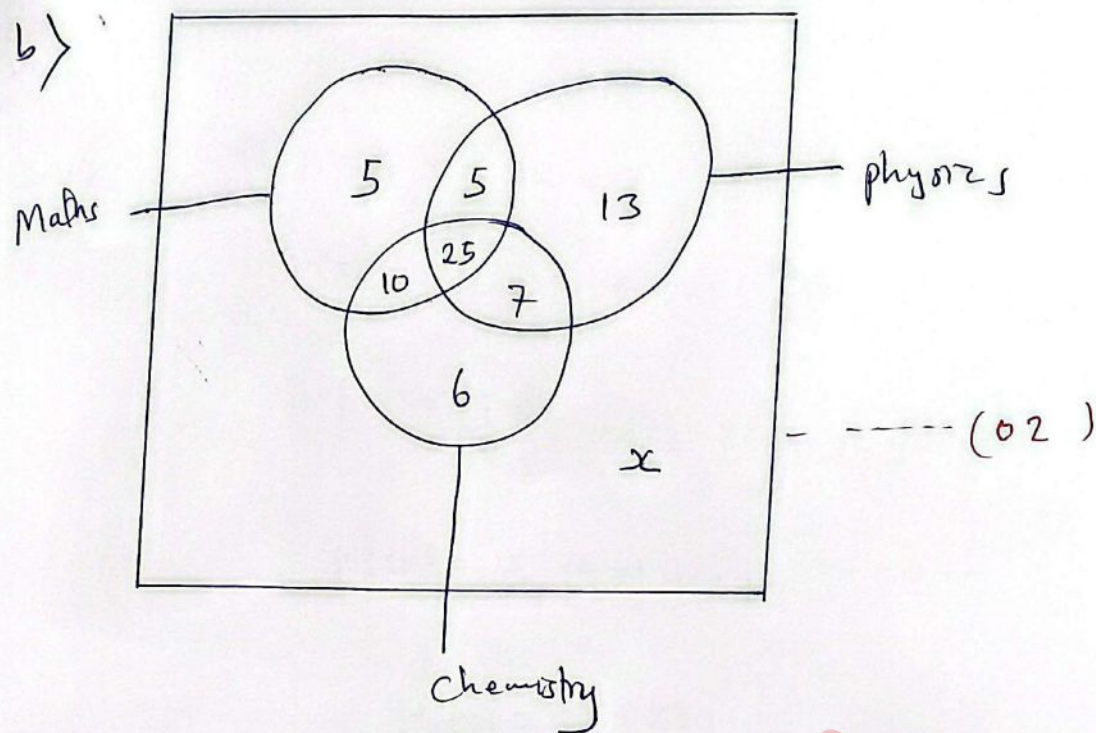
$$\begin{aligned}
 5. a) 1) (P - Q) &= (P \cup Q') \quad \text{--- Given} \\
 &= (P \cap Q') \cap (P \cup Q')' \quad \text{--- Definition } A - B = A \cap B' \\
 &= (P \cap Q') \cap (P' \cap Q) \quad \text{--- De Morgan's law} \\
 &= (P \cap P') \cap (Q' \cap Q) \quad \text{--- Associative law} \\
 &= \phi \cap \phi \quad \text{--- Complement law} \\
 &= \phi \quad \text{--- Idempotent,} \\
 &\quad \quad \quad (03)
 \end{aligned}$$

$$\therefore (P - Q) = (P \cup Q') = \phi$$

$$\begin{aligned}
 (ii) [A \cap (A \cup B)']' &\quad \text{--- Given} \\
 &= A' \cup (A \cup B) \quad \text{--- De Morgan's law} \\
 &= (A' \cup A) \cup B \quad \text{--- Associative law} \\
 &= 1 \cup B \quad \text{--- Complement law} \\
 &= 1 \quad \text{--- Identity law} \\
 &\quad \quad \quad (03) \\
 \therefore [A \cap (A \cup B)']' &= 1
 \end{aligned}$$

20/31

5 6 >



1) Number of students who failed at least one subject

$$= 5 + 5 + 13 + 10 + 25 + 7 + 6 = 71 \text{ students. (01)}$$

11) Students who passed the exam

$$X + 71 = 150$$

$$X = 150 - 71$$

$$X = 79$$

----- (01)

$\therefore$ Student who passed exam = 79 students

14/3x

6

a) Given

$$1) \quad f(x) = \frac{9}{5}x + 32.$$

$$g(x) = \frac{5}{9}(x - 32) + 273. \quad \dots \dots \dots 01$$

$$g \circ f(x) = \frac{5}{9} \left(\left(\frac{9}{5}x + 32 \right) - 32 \right) + 273. \quad \dots \dots \dots 01$$

$$g \circ f(x) = \frac{5}{9} \left(\frac{9}{5}x \right) + 273.$$

$$g \circ f(x) = x + 273 \quad \dots \dots \dots 01$$

$$11) \quad \text{at } 25^\circ\text{C} \Rightarrow x = 25^\circ\text{C}$$

$$\text{from } g \circ f(x) = x + 273 \quad \dots \dots \dots 01$$

$$g \circ f(25) = 25 + 273 \quad \dots \dots \dots 01$$

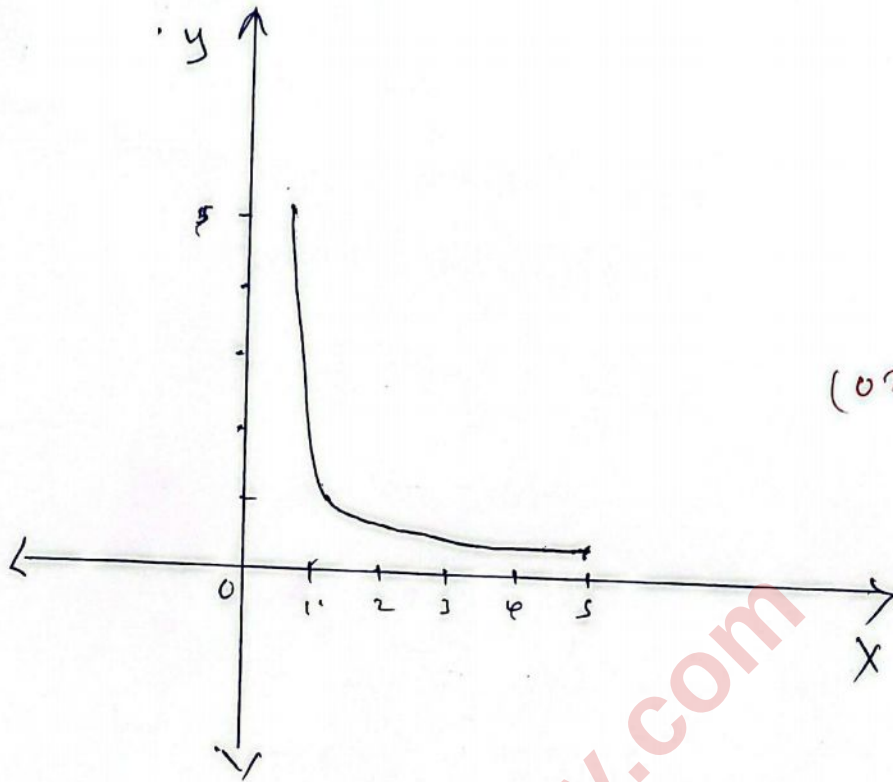
$$= 298 \text{ K}.$$

$\therefore$ The kelvin temperature is 298K. $\dots \dots \dots 01$

12/31

6 b)

$\Rightarrow f(x) = \frac{1}{x}$ for $0 < x \leq 5$.



(ii) Range = $\{y \in \mathbb{R} : \frac{1}{5} \leq f(x) \leq \infty\}$ 01

(iii) The mapping is one to one function 01

13/31

a) Given

$$u = \frac{5xy^2}{z^3}$$

Apply $\ln$ both side.

$$\ln u = \ln 5 + \ln x + 2 \ln y - 3 \ln z. \quad \dots \text{only } 2$$

Differentiate both side.

$$\frac{du}{u} = \frac{dx}{x} + 2 \frac{dy}{y} - 3 \frac{dz}{z}, \quad \dots \text{only } 2$$

for maximum error

$$\frac{\Delta u}{u} = \frac{\Delta x}{x} + 2 \frac{\Delta y}{y} - 3 \frac{\Delta z}{z}, \quad \dots \text{only } 2$$

but

$$u = \frac{5xy^2}{z^3} = \frac{5(1)(1)^2}{(1)^3}$$

$$= 5, \quad \dots \text{only } 2$$

$$\frac{\Delta u}{5} = \frac{0.001}{1} + 2 \frac{(0.001)}{1} + 3 \frac{(0.001)}{1}$$

$$\frac{\Delta u}{5} = 0.006,$$

$$\Delta u = 0.03,$$

$\therefore$ Maximum error in u is 0.03 only 2

(14/31)

6) $\Rightarrow$ Given $f(x) = x^4 - 4x + 1$

For the interval 0 to 1

$$f(0) = 1$$

$$f(1) = 1 - 4 + 1$$

$$= -2$$

$f(x) = 0$ has a real root b/n 0 and 1 if

$$f(0) \cdot f(1) < 0$$

$$f(0) \cdot f(1) = (1)(-2)$$

$$= -2$$

$\therefore f(0) \cdot f(1) = -2$ which is less than 0.

therefore $f(x) = 0$ has the root between 0 and 1

ii) $f(x) = x^4 - 4x + 1$

$$f'(x) = 4x^3 - 4$$

from

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^4 - 4x_n + 1}{4x_n^3 - 4}$$

$$= \frac{4x_n^4 - 4x_n^4 + 4x_n^3 - x_n^4 + 4x_n - 1}{4x_n^3 - 4}$$

$$x_{n+1} = \frac{3x_n^3 - 1}{4x_n^3 - 4}$$

(15/3)

for

$$X_1 = \frac{3X_0^4 - 1}{4X_0^3 - 4}$$

for $X_0 = 0.2$

$$X_1 = 0.2508 \quad \text{--- only } 2$$

$$X_2 = \frac{3X_1^4 - 1}{4X_1^3 - 4}$$

$$X_2 = 0.2541 \quad \text{--- only } 2$$

$$X_3 = \frac{3X_2^4 - 1}{4X_2^3 - 4}$$

$$X_3 = 0.2541 \quad \text{--- only } 2$$

$\therefore$ The root is 0.2541.

7 c) Let

$$f(x) = e^{x^2}$$

$$h = \frac{1-0}{4} = 0.25 \quad \text{--- only } 2$$

x	0.00	0.25	0.50	0.75	1.00
f(x)	1.0000	1.645	1.2840	1.7551	2.7183

--- 01

16/5

c)

Run

$$\int_a^b f(x) dx = \frac{h}{2} \left[\text{sum of extreme ordinate} + 2 (\text{sum of middle ordinate}) \right]$$

$$\int_0^1 e^{x^2} dx = \frac{0.25}{2} \left[3.7183 + 2(4.1036) \right] \dots \dots 01$$

$$= 1.4907$$

$$\therefore \int_0^1 e^{x^2} dx \approx 1.4907$$

only 2

8 a)

Given A(1, 2) and B(4, 6).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{6 - 2}{4 - 1} = \frac{4}{3} \dots \dots \text{only } 2$$

~~Its eqn from~~

Its eqn will be

$$y = \frac{4}{3}(x-1) + 2$$

$$3y = 4x - 4 + 6 \dots \dots \text{only } 2$$

$$\left(\frac{17}{3} \right)$$

8 a)

$$3y = 4x + 2$$

$$4x - 3y + 2 = 0 \quad \text{--- --- --- only 2}$$

Distance from (2,3) to the line $4x - 3y + 2 = 0$

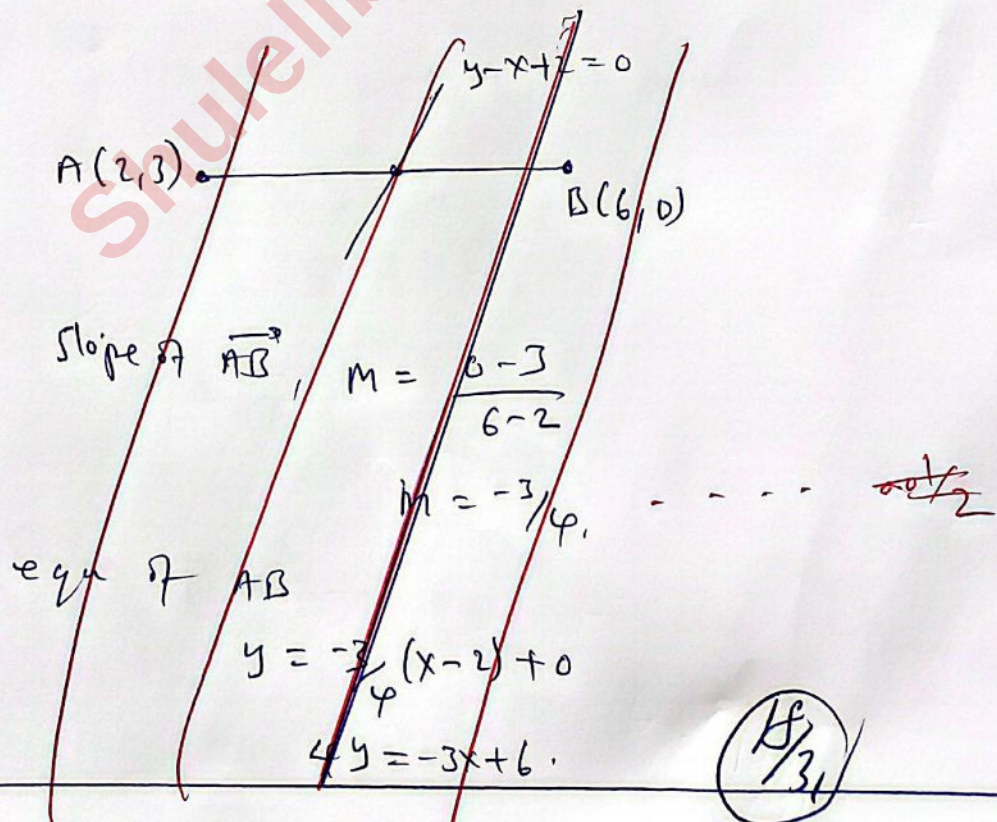
$$d = \left| \frac{4(2) - 3(3) + 2}{\sqrt{4^2 + 3^2}} \right| \quad \text{--- --- --- only 2}$$

$$d = \left| \frac{-1 + 2}{\sqrt{25}} \right| \quad \text{--- --- --- only 2}$$

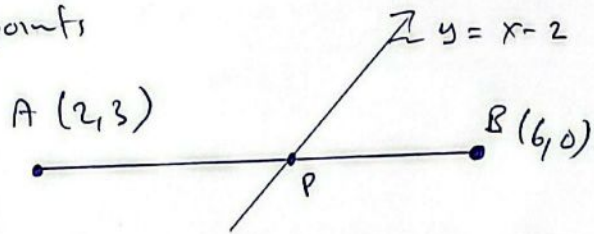
$$d = \frac{1}{5} \text{ units.} \quad \text{--- --- --- only 2}$$

$\therefore$ The perpendicular distance is $\frac{1}{5}$ units.

b)



8) Given points



$$\text{Slope of } AB, m = \frac{0-3}{6-2}$$

$$m = -\frac{3}{4} \dots \dots \dots \text{only}_2$$

Equation of line AB will be

$$y = -\frac{3}{4}(x-2) + 3 \dots \dots \dots \text{only}_2$$

$$4y = -3x + 6 + 12$$

$$4y = -3x + 18.$$

$$3x + 4y = 18 \dots \dots \dots \text{only}_2$$

Solving for point of intersection P.

$$\begin{cases} x - y = 2 \\ 3x + 4y = 18 \end{cases}$$

So

$$(x, y) = \left(\frac{26}{7}, \frac{12}{7}\right) \dots \dots \dots \text{only}_2$$

Since P divide AB internally then

$$\left(\frac{19}{3}\right)$$

8 b) $P(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$

$$\left(\frac{26}{7}, \frac{12}{7} \right) = \left(\frac{6m+2n}{m+n}, \frac{0m+3n}{m+n} \right) \dots \text{only } 2$$

$$\left(\frac{26}{7}, \frac{12}{7} \right) = \left(\frac{6m+2n}{m+n}, \frac{3n}{m+n} \right)$$

$$\frac{26}{7} = \frac{6m+2n}{m+n} \quad \text{or} \quad \frac{12}{7} = \frac{3n}{m+n} \dots \text{only } 2$$

$$12(m+n) = 21n \dots \text{only } 2$$

$$12m + 12n = 21n$$

$$12m = 21n - 12n$$

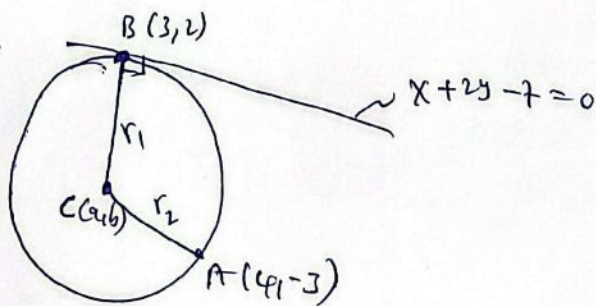
$$12m = 9n$$

$$\frac{m}{n} = \frac{9}{12} = \frac{3}{4}$$

$$\therefore m:n = 3:4 \dots \text{only } 2$$

20/11

8 c) consider



Slope of tangent line $X + 2Y - 7 = 0$ is $-\frac{1}{2}$.

Since $\overline{BC}$ is $\perp$ to tangent line

Slope of $\overline{BC} = 2$ ----- only $\frac{1}{2}$

eqn of $\overline{BC}$

$$y = 2(x - 3) + 2$$

$$y = 2x - 6 + 2$$

$$y = 2x - 4$$

at (a, b) - centre

$$2a - b = 4 \quad \text{--- (1) --- only } \frac{1}{2}$$

from

$$r_1 = r_2$$

$$(a - 3)^2 + (b - 2)^2 = (a - 4)^2 + (b + 3)^2$$

$$a^2 - 6a + 9 + b^2 - 4b + 4 = a^2 - 8a + 16 + b^2 + 6b + 9$$

$$2a - 10b = 12 \quad \text{--- (11) --- only } \frac{1}{2}$$

$$\left(\frac{24}{13}, \frac{10}{13} \right)$$

6

Solve (i) and (ii)

$$2a - b = 4$$

$$2a + 10b = 12$$

$$a = \frac{14}{9}, \quad b = -\frac{8}{9}, \quad \dots \dots \dots \text{only } 2$$

from

$$r^2 = (a-3)^2 + (b-2)^2$$

$$r^2 = \left(\frac{14}{9} - 3\right)^2 + \left(-\frac{8}{9} - 2\right)^2$$

$$r^2 = \frac{845}{81}$$

from

equation of circle

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\left(x - \frac{14}{9}\right)^2 + \left(y + \frac{8}{9}\right)^2 = \frac{845}{81}$$

$$9x^2 + 9y^2 - 28x + 16y - 65 = 0$$

∴ The eqn of circle is

$$9x^2 + 9y^2 - 28x + 16y - 65 = 0$$

22/13

Q. (a) $\int \frac{1}{x^3 - 3x^2 + 2x} = \int \frac{1}{x(x-2)(x-1)}$

$$\frac{1}{x(x-2)(x-1)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x-1} \quad \text{--- (i)}$$

$$= \frac{A(x-2)(x-1) + Bx(x-1) + Cx(x-2)}{x(x-2)(x-1)}$$

$$1 = Ax^2 - 3Ax + 2A + Bx^2 - Bx + Cx^2 - 2Cx$$

$$= (A+B+C)x^2 + (-3A-B-2C)x + 2A$$

$$2A = 1, \quad A = \frac{1}{2} \quad \text{--- (ii)}$$

$$A+B+C = 0$$

$$-3A - B - 2C = 0$$

$$B + C - \frac{1}{2} = 0$$

$$B + C = \frac{1}{2} \quad \text{--- (iii)}$$

$$-2A - B - 2C = \frac{3}{2} \quad \text{--- (iv)}$$

$$(i) - (iii)$$

$$B = \frac{1}{2}, \quad C = -1$$

23/31

$$\int \left(\frac{A}{2x} + \frac{B}{2x-2} + \frac{C}{x-1} \right) dx$$

$$= \int \left(\frac{2}{x} + \frac{2}{x-2} + \frac{1}{x-1} \right) dx \quad \dots \dots \dots 01$$

$$= \frac{1}{2} \ln x + \frac{1}{2} \ln (x-2) - \ln (x-1) + C \quad \dots \dots \dots \text{sol } 2$$

$$\text{b) } \int_0^{\pi/4} \frac{1}{1 + \cos^2 x} dx$$

$$= \cos^2 x = \frac{1 + \cos 2x}{2}$$

$$= \int_0^{\pi/4} \frac{2}{2 + 1 + \cos 2x} dx = \int_0^{\pi/4} \frac{2}{3 + \cos 2x} dx \quad \dots \dots \dots 01$$

$$\text{let } t = \tan x, \quad \cos 2x = \frac{1-t^2}{1+t^2}$$

$$dt = \sec^2 x dx$$

$$3 + \cos 2x = 3 + \frac{1-t^2}{1+t^2} = \frac{4+t^2}{1+t^2} \quad \dots \dots \dots \text{sol } 2$$

24/24

$$b) \frac{1}{2} \int_0^{\pi/4} \frac{1}{2+t^2} dt$$

$$t = \sqrt{2} \tan \theta$$

$$dt = \sqrt{2} \sec^2 \theta d\theta \quad \text{--- } \theta = \pi/4$$

$$\frac{1}{2} \int_0^{\pi/4} \frac{\sqrt{2} \sec^2 \theta}{2(1+\tan^2 \theta)} d\theta = \frac{\sqrt{2}}{4} \int_0^{\pi/4} d\theta = \frac{\sqrt{2}}{4} \theta \Big|_0^{\pi/4}$$

$$\theta = \tan^{-1} \frac{t}{\sqrt{2}} \quad \theta = \tan^{-1} \left(\frac{\tan x}{\sqrt{2}} \right) \quad \text{--- } 0$$

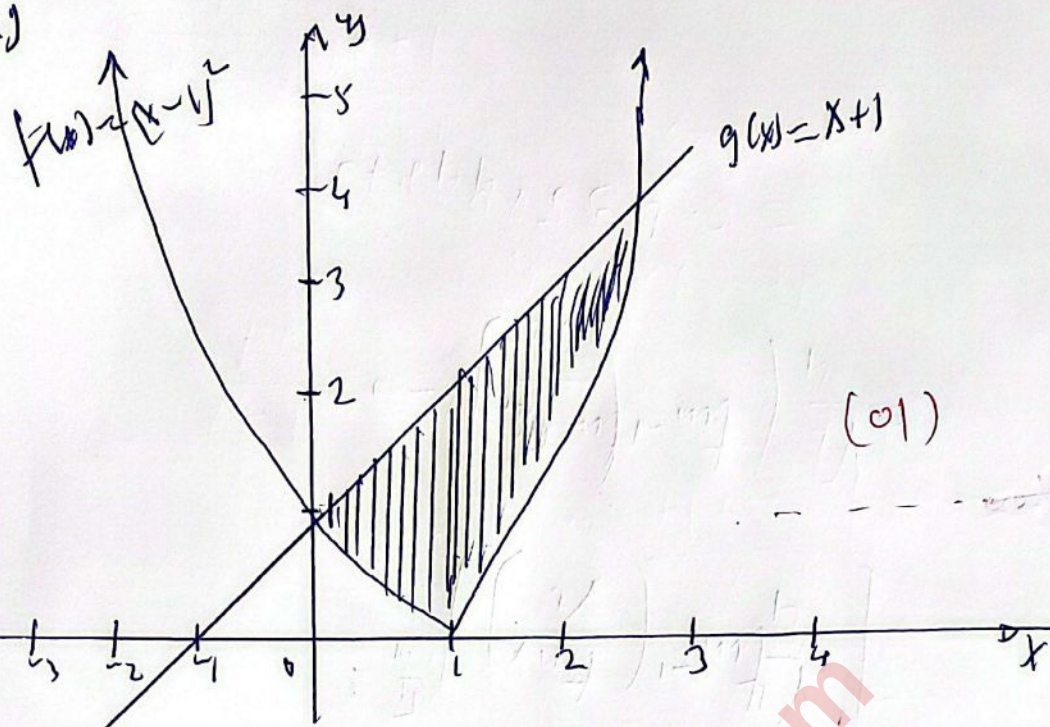
$$= \left[\frac{\sqrt{2}}{4} \tan^{-1} \left(\frac{\tan x}{\sqrt{2}} \right) \right]_0^{\pi/4}$$

$$= \frac{\sqrt{2}}{4} \left(\tan^{-1} \left(\frac{\tan \pi/4}{\sqrt{2}} \right) - 0 \right)$$

$$= 0.4352099875 \quad \text{--- } 0$$

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(c)



$$A = \int_1^3 (x+1) - (x-1)^2 dx$$

$$\int_1^3 (x+1 - x^2 + 2x - 1) dx$$

$$\int_1^3 (3x - x^2) dx$$

$$\left[\frac{3x^2}{2} - \frac{x^3}{3} \right]_1^3$$

$$\frac{3(3)^2}{2} - \frac{3^3}{3} = 0 \quad \text{--- 9 sq. unit}$$

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Q1) $x = t^2 + 2t$, $y = t^3 - t$

$$\frac{dx}{dt} = 2t + 2, \quad \frac{dy}{dt} = 3t^2 - 1$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2 - 1}{2t + 2}$$

$$\frac{dy}{dx} = \frac{3t^2 - 1}{2t + 2}$$

Q2) $y = \frac{1}{2} \sin x$

(i) slope $= \left(\frac{dy}{dx} \right)_{t=1} = \frac{3(1)^2 - 1}{2(1) + 2} = \frac{2}{4}$

$$m = \frac{1}{2}$$

The point $x = t^2 + 2t$, $y = t^3 - t$

$$x = 3, \quad y = 0$$

$$(3, 0)$$

$$\left(\frac{2}{3}, \frac{1}{3} \right)$$

$$m = \frac{1}{2}, \quad (3, 0)$$

$$y = \frac{1}{2}(x-3) + 0$$

$$y = \frac{1}{2}x - \frac{3}{2} \quad \text{--- -- -- -- -- } 0$$

$$b) \quad y = \frac{1}{2} \sin 2x$$

$$f(x+h) = \frac{1}{2} \sin 2(x+h) \quad \text{--- -- -- } \sin \frac{1}{2}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{1}{2} \sin 2(x+h) - \frac{1}{2} \sin 2x}{h} \quad \text{--- -- -- } \sin \frac{1}{2}$$

$$= \lim_{h \rightarrow 0} \frac{\sin [2x+2h] - \sin 2x}{h}$$

$$= \frac{1}{2} \lim_{h \rightarrow 0} \frac{\sin 2x \cos 2h + \cos 2x \sin 2h - \sin 2x}{h}$$

$$= \frac{1}{2} \lim_{h \rightarrow 0} \frac{\sin 2x \cos 2h - \sin 2x + \sin 2x \cos 2h}{h}$$

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$$= \frac{1}{2} \lim_{h \rightarrow 0} \frac{\sin 2x (\cos h - 1) + \sin h \cos x}{h}$$

$$= \frac{1}{2} \lim_{h \rightarrow 0} \sin 2x (\cos h - 1) + \cos x \cdot \frac{\sin h}{h}$$

$$\frac{1}{2} \cos 2x \cdot 2 = \cos 2x$$

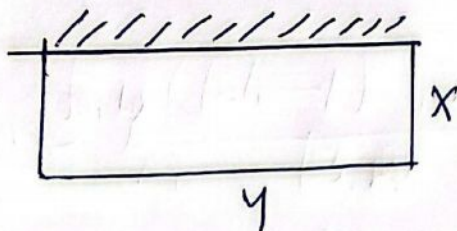
$$= \cos 2x$$

Q1

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0 (c)



$$\text{total length} = 2x + y$$

$$2x + y = 120$$

$$y = 120 - 2x \quad \text{---} \quad \text{only } 2$$

$$A = xy$$

$$A = x(120 - 2x) \quad \text{---} \quad \text{only } 2$$

$$A = 120x - 2x^2$$

$$\frac{dA}{dx} = 120 - 4x \quad \text{---} \quad \text{only } 2$$

$$120 - 4x = 0$$

$$4x = 120$$

$$x = 30 \quad \text{---} \quad \text{only } 2$$

$$y = 120 - 2 \times 30$$

$$y = 60 \quad \text{---} \quad \text{only } 2$$

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Maximum area = 30×60
Area of the field = 1800 m^2 ----- $\frac{100}{2}$

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