PRIME MINISTERS OFFICE

REGIONAL ADMINISTRATION AND LOCAL GOVERNMENT

HIGH HOPE SECONDARY SCHOOLS

FORM SIX MID-TERM EXAMINATION

ADVANCED MATHEMATICS 1

TIME: 3HOURS

INSTRUCTIONS: **AUGUST 2026**

1. This paper consists of NINE (9) questions, with a total of 100% marks.
2. Answer all questions.
3. All necessary working and answers for each question done must be shown clearly.
4. Mathematical tables and non-programmable calculators may be used.
5. Cellular phones and any unauthorized materials are not allowed in examination room.
6. Write your examination number on every page of your answer booklet(s

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1. a) Prove that $\cosh^{-1}x = \frac{1}{2} \ln \left(\frac{x+1}{x-1}\right)$ for $x > 0$

b) . Given $y = \cosh^{-1}\left[\frac{1+x^2}{1-x^2}\right]$, find $\frac{dy}{dx}$ and simplify your answer

c). Solve the simultaneous equations giving your answer in logarithmic form

$$\begin{cases} Cosh\, x - 3Sinhy = 0 \\ 2Sinhx + 6Coshy = 0 \end{cases}$$

2. a) A manufacturer of jams has 720 kg of strawberry syrup and 800kg of mango syrup for making two types, each type is made by mixing-strawberry and mango as follows. Grade A: 60% strawberry and 40% mangoes. Grade B: 30% strawberry and 70% mango. The jam is sold in 400gram jars. The selling prices are as follows. Grade A is 48/= per jar, while Grade B is 30/= per jar. Find how many of each type the manufacturer makes to maximise profit.

b) A wheat flour company has factories of A and B which supplies warehouse of C and D. The weekly factory capacities are 160 and 140 tons respectively and were houses requirements are 70 and 120 tons respectively. The cost of transportation of one tone of wheat flour from A to C is Tsh 160 and cost from A to D is Tsh 240 similarly the transport cost from B to C is Tsh 200 and from B to D is Tsh 260. Formulate a mathematical model of the following

i. The objective function to be minimised by the company so as to supply tones of wheat flour to each warehouse.

ii. The inequalities associated with the transportation problem

iii. Does the problem balance or not, why?

3. a) The height X cm of a sample of 80 female is summarised by the equation $\Sigma(X-160) = 240$ and $\Sigma(X-160)^2 = 8720$. Find the standard deviation of the height of the female students.

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b). The scores of advanced mathematics test of 200 students at a certain school were recorded as follows

Interval	30-40	40-50	50-60	60-70	70-80	80-90
Number of students	20	41	48	43	30	18

Estimate the mean and standard deviation using coding method (Use assumed mean, A = 55).

4. a) Use laws of algebra of set to simplify the following

i. $[A' \cup (A \cap B')]'$

ii. $(A \cap B) \cup (A \cap B') \cup (A' \cap B)$

b). In a survey of 100 students, it was found that 50 used college library books, 45 had their own books and 30 used borrowed books, 20 used college library and their own books only 15 used their own books and borrowed books only, whereas 10 used college library books and borrowed books. Only and 5 used all the three sources.

i. How many did not use any of the three sources?

ii. How many used their books only?

iii. How many used one source of books only?

5. a). Given that $g(x) = 2x^2 - 7x$ and h(x) = 3x−5 find

i. goh(x)

ii. Sketch the graph of goh(x)

b). A certain Biometric device uses natural numbers in its functions as B (x) = $\frac{x}{x^2-4}$

i. What are the values such that a device cannot give an

ii. Represent the B (x) on your graph and show all important features

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6.a) The function is derived as $f(x) = x^2 - a$, show that $X_{n+1} = \frac{1}{2}[xn + \frac{a}{xn}]$

b). The ball seems to float on the water such that its equation gives depth x to which the ball is submerged under water is given by $x^3 - 0.165x^2 + 3.993 \times 10^{-3}$ (Depth in meters) use the Newton-Raphson method to find.

i. The depth x to which the ball is submerged under water. Conduct interactions (Three iterations) to estimate the root of the above equation.

ii. The absolute error at the end of each iteration to 4 significant figures

7.a) Find the perpendicular distance from the centre of the circle $2x^{-2}+2y^2 +8x+12y - 136=0$ to the line $12x + 16y + 12=0$

b). At what angle are the lines whose equations are $ax + by + c = 0$ and $(a - b)x + (a + b) + y + d = 0$ inclined to each other?

c). Find the equation of a circle concentric with the circle $2x^2 + 2y^2 + 8x + 10y - 39 = 0$ and having its area equal to $2x$

8. (a). if $\int_0^{\frac{\pi}{2}} x\sin x\, dx = \int_0^2 (ax^2 + 2x)\, dx$ find the value of a.

(b). integrate the following with respect to x: $\frac{x}{2x^2-x+1}$

(c). Find the volume generated by revolving about x- axis the area cut off from the parabola $9y = 4(9 - x^2)$ by the line $4x+3y = 12$ correct to two significant figures.

9. a). Given that $y = \theta - \cos\theta$ and $x = \sin\theta$ show that $\frac{d^2y}{dx^2} = \frac{1+\sin\theta}{\cos^3\theta}$

b). If $x^p y^q = (x + y)^{p+q}$, prove that $\frac{dy}{dx} = \frac{y}{x}$

c). Find the value of the constant "a'' and "b'' of a point of inflexion on the graph $y = ax^3+bx^2+2$ is (1,0). Hence find the maximum and minimum points of the graph.

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