THE UNITED REPUBLIC OF TANZANIA
PRESIDENT'S OFFICE
REGIONAL ADMINISTRATION AND LOCAL GOVERNMENT
FORM SIX MOCK EXAMINATIONS
TANGA REGION

CODE: 142/2 **ADVANCED MATHEMATICS 2**

TIME: 3 HOURS **Thursday,3rd Dwcember 2020A.M**

INSTRUCTIONS

1. This paper consists of section **A** and **B** with total of **eight (8)** questions
2. Answer all questions in section **A** and **ANY** two (2) questions from section **B**
3. All work done and answers for each questions done must be shown clearly
4. Mathematical table and non – programmable calculators may be used
5. Cellular phones are not allowed in examination room
6. Write your examination number on every page of your answer booklet(s)

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SECTION A (60 MARKS)

1.(a) Events A and B are such that $P(A)=\frac{1}{3}$, $P(B/A)=\frac{1}{4}$ and $P(\overline{B}/\overline{A})=\frac{4}{5}$.

By drawing a tree diagram, or otherwise, find $(i)\ P(B)\ (ii)\ P(A\cup B)$

(b) A man is equally likely to choose among the three routes from his house to the railway station. It is Known from the experience that the probability of missing a train by choosing routes A, B and C are 1/5, 2/5 and 3/5 respectively. A man sets a root on a dry day and missed a train. Find the probability that the route chosen was C

(c) (i) If the random variable X is such that $X \sim Bin(n, p)$then derive the Mean.

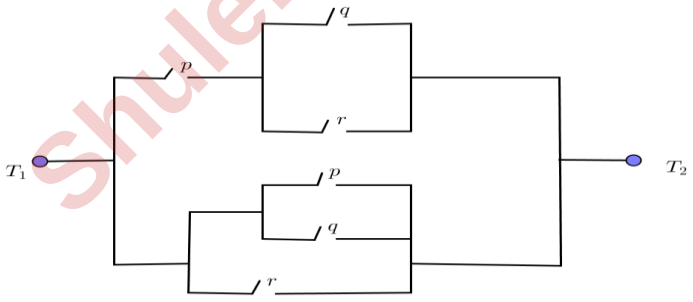
(ii) A box contains a large number of red and yellow tulip bulbs in the ratio $1:3$. Bulbs arepicked at random from the box. How many bulbs must be picked so that the probability that there is at least one red tulip bulb among them is greater than 0.95?

(d) The continuous $r.v.X$ has $p.d.f.\ f(x)$ where $f(x)=\frac{3}{4}(1+x^2),\ 0\le x\le 1$.

If $E(X)=\mu$ and $Var(X)=\sigma^2$, find $P(|X-\mu|<\sigma)$.(correct to 3 significant figures)

2. (a) Use laws of algebra of proposition to show whether the proposition $(p\leftrightarrow q)\rightarrow(p\rightarrow q)$is or is not a tautology.

(b) Draw a simplified diagram for a network that corresponds to the following electrical network.



(c) Use the truth table to test the validity of the following argument:

"If the computer program is incorrect, it will neither produce correct output for a given test data nor terminate the data. This computer program produces the correct output for a given data. Therefore, this computer program is correct.

3. (a) (i) Given that $\underline{V} = a\underline{i} + b\underline{j} + c\underline{k}$, Simplify the expression $|\hat{\underline{V}}|$

(ii) Find a vector of magnitude 9 which is perpendicular to both vectors $\underline{a} = 4\,\underline{i} - \underline{j} + 3\,\underline{k}$ and $\underline{b} = -2\,\underline{i} + \underline{j} - 2\,\underline{k}$,

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(b) (i) Find the area of a triangle formed by parallelogram having the diagonals

$\underline{a} = 3\underline{i} + \underline{j} - 2\underline{k}$ and $\underline{b} = \underline{i} - 3\underline{j} + 4\underline{k}$

(ii) Derive the sine rule by vector theorem.

(c) The acceleration of a body at any time t is given by $\underline{a}(t) = 2t\underline{k}$, Find

(i)The velocity of the body after 3 sec if when ,$t = 0$ it's velocity is $\underline{v}(t) = 2\underline{i} - 5\underline{k}$.

(ii) The displacement of the car covered after 3 sec if when ,$t = 0$ it's displacement is $\underline{r}(t) = 9\underline{i} + 6\underline{j} + 3\underline{k}$.

4.(a) Use the infinite series for $\cos\theta$, $\sin\theta$ and e^x to show that $z = re^{i\theta}$ and hence express $e^{\frac{\pi}{2}i}$ as a purely imaginary number.

(b) Find the fourth root of 16 in the form $a + bi$.

(c) If x, y, a and b are real numbers and if $x + yi = \dfrac{a}{b + \cos\theta + i\sin\theta}$.

Show that $(b^2 - 1)(x^2 + y^2) + a^2 = 2abx$.

(d) If $z_1 = 3 + i$, $z_2 = -3 - i$ and $z = x + yi$ determine the locus of the set of points $P(x, y)$ on the Argand diagram given that $|z - z_1| = |z - z_2|$

SECTION B (40 MARKS)

Answer any two (2) questions from this section. Extra questions will not be marked.

5. (a) Find the value of $\sin^2 A\operatorname{cosec}\left(\frac{\pi}{2} - A\right) - \cot^2\left(\frac{\pi}{2} - A\right)\cos A$.

(b) Solve equation $\cos 2x + \sqrt{3}\sin 2x = 2\cos x, \quad 0 \le x \le 2\pi$.

(c) (i) Prove that $\tan^{-1}\left(\frac{a+x}{a-x}\right) - \tan^{-1}\left(\frac{x}{a}\right) = \frac{\pi}{4}$.

(ii) By using the result of 5(c)(i) above, solve the equation

$$\tan^{-1}\left(\frac{1+x}{1-x}\right) - \frac{1}{2}\tan^{-1} x = \tan^{-1}\left(\sqrt{3}\right)$$

(d) Given that $y = \dfrac{\sin x - 2\sin 2x + \sin 3x}{\sin x + 2\sin 2x + \sin 3x}$, prove that $y = -\tan^2\left(\frac{x}{2}\right)$. Hence, find the exact value of $\tan^2 15^0$ in the form $p + q\sqrt{r}$ where p, q and r are integers.

6. (a) Express $\dfrac{x^2 + 2x + 18}{x(x^2 + 3)^2}$ in partial fractions.

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(b) Prove by mathematical Induction that $\sum_{r=1}^{n}\frac{r}{2^r}=2-\left(\frac{1}{2}\right)^n(2+n)$

for all positive value of n

(c) Given that the first three terms in the expansion in ascending power of x of $(1-8x)^{\frac{1}{4}}$ are the same as the first three terms in the expansion of $\left(\frac{1+ax}{1+bx}\right)$,

find thevalues of a and b. Hence, find an approximation to $(0.6)^{\frac{1}{4}}$ in the form $\frac{p}{q}$ where p and q are integers, $q\neq 0$.

(d) Find the relation between a, b, c and d if one root of the equation $ax^3+bx^2+cx+d=0$ is equal to the sum of the other two.

7. (a) Form the differential equation of the family of circle touching the $y-$axis at origin.

(b) Find the particular solution of the differential equation $(x-y)\frac{dy}{dx}=(x+2y)$,

given that $y=0$ when $x=1$.

(c) Solve $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+5y=0$, $y=2$ and $\frac{dy}{dx}=\frac{d^2y}{dx^2}$ when $x=0$.

(d) The rate at which radioactive substance decays is proportional to the number of such nuclei that are present at the time in a given sample.In a certain sample 10% of the original number of radioactive nuclei have undergone disintegration in a period of 100years. Find the percentage of the original radioactive nuclei that will remainafter 1000 years.

8. (a) Find the center, eccentricity, foci and directrix of $4x^2+9y^2-16x+72y+124=0$

(b) The point $P(at_1^2,2at_1)$ and $Q(at_2^2,2at_2)$ on the parabola $y^2=4ax$ are such that PQ passes through the focus of the parabola. Show that the locus of the midpoint of PQ is another parabola and find the coordinates of its focus.

(c) Find the equations of the tangents to the hyperbola $x^2-9y^2=9$ that are drawn from the point $(3,2)$ and find the points of contact.

(d) (i) If a polar equation of a curve is $r^2=4\sin 2\theta$ find the Cartesian equation

(ii) Draw a sketch of the curve $r=1-2\cos\theta$.

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