THE UNITED REPUBLIC OF TANZANIA
PRESIDENT'S OFFICE, REGIONAL ADMINISTRATION AND LOCAL GOVERNMENT

FORM SIX SPECIAL SCHOOLS JOINT EXAMINATION

CODE: 142/1 **ADVANCED MATHEMATICS 1**

Time: 3:00 HRS Wednesday **28-February-2024 AM**

INSTRUCTIONS

1. This paper consists of ten (10) questions.
2. Answer **ALL the** questions. Each question carries **ten (10)** marks.
3. All necessary working and answers of each question done must be shown clearly.
4. Mathematical tables and non-programmable calculators may be used
5. All writing must be in **blue** or **black** ink except drawing which must be in pencil.
6. Cellular phones and any unauthorized materials are not allowed in examination room
7. Write your **Examination Number** on every page of your answer sheet.

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1. By using non- programmable scientific calculator to;

a) Evaluate $\log_3 \dfrac{\left(\begin{vmatrix}1 & 2 & 3\\ 0 & 1 & 3\\ 2 & -1 & 5\end{vmatrix}\right) - \left(\ln\left|\left(\frac{3}{13}\right)^{\frac{1}{3}}\right|\right)\sin\left(\frac{-\pi}{6}\right)}{\int_{-2}^{3}(x-2)(x+1)(x-1)dx}$ (to 4 decimal places)

b) find angle θ in degrees, minutes and seconds, if $\tan\theta = \dfrac{^{12}C_8 \times {}^{10}P_2 \times \sin^{-1}(0.8695)}{\sqrt[3]{831} \times \tan 15^\circ \times \ln(18.62)}$

c) Calculate modulus and argument of the complex number Z if $Z = \dfrac{(3-i)(2+3i)}{3+i}$

2. a) Prove that $\cot^{-1} x = \frac{1}{2}\ln\left\{\frac{1+x}{x-1}\right\}$ for $x > 0$

b) Given $y = \cosh^{-1}\left(\dfrac{1+x^2}{1-x^2}\right)$, *find* $\dfrac{dy}{dx}$ and simplify your answer

c) Solve the simultaneous equations, giving your answer in logarithmic form

$$\begin{cases} \cos hx - 3\sin hy = 0 \\ 2\sinh x + 6\cos h\, y = 5 \end{cases}$$

3. A farmer owns two godowns G1 and G2 containing 350 bags and 250 bags of rice respectively. Three customers C1, C2 and C3 have ordered 200 bags , 150 bags and 250 bags respectively. The cost per bag of transporting rice to each customer from each godown are as follows.

	Transport cost		
	C1	C2	C3
Godown G1	300/=	600/=	600/=
Godown G2	500/=	400/=	500/=

The farmer wishes to meet the orders at a minimum cost.

a) How should he supply his customers?
b) What is the overall minimum transport cost?

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4. (a). The mean and standard deviation of 20 numbers are 8 and 3 respectively while the mean and standard deviation of another set of 30 numbers are 6 and 2 respectively.

Determine the mean and standard deviation of the combined set of 50 numbers.

(b) The scores of Advanced Mathematics1 test of 200 students at a certain school were recorded as follows

Marks	30--40	40—50	50--60	60—70	70--80	80--90
Number of students	20	41	48	43	30	18

Estimate the mean and standard deviation using coding method, **(Use Assumed Mean, A=55).**

5. a) If set A and B are defined by $A = \{x \in \mathbb{R}: -1 \leq x \leq 2\}$ and $B = \{x \in \mathbb{R}: 2 \leq x < 5\}$, find $A - B$ in the same notation.

b) Using the laws of algebra of sets, simplify the following expression.

$(P \cap Q') \cup (P' \cup Q')$

c) In a class of 17 girls and 15 boys, 22 play handball, 16 play basketball, 12 of boys' play handball, 11 of the boys play basketball, 10 of the boys play both basketball and handball, 3 of the girls play neither of the two games. Use a Venn diagram to determine.

(i) the number of girls who play both games
(ii) the number of participants who play at least one game

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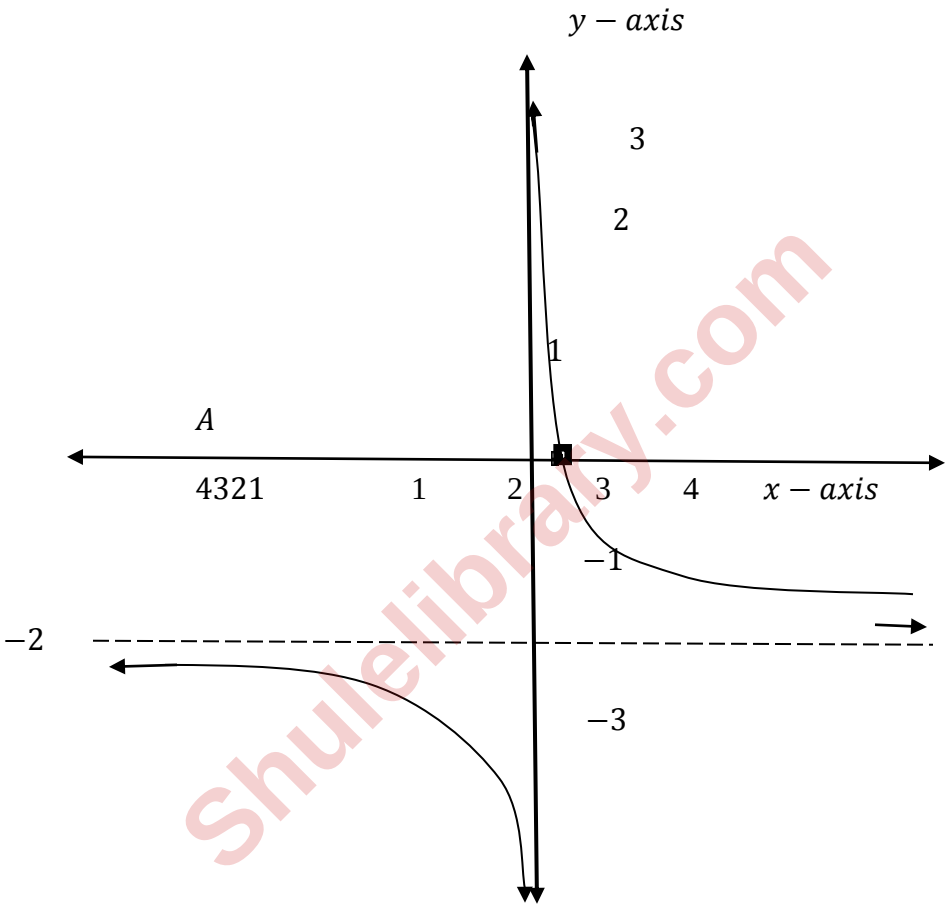
6. . (a) If $f:x\to 5x+4$ *and* $g:x\to 6x-k$, determine the value of k for which for which $f\circ g(x)=g\circ f(x)$.

(b). Figure below shows that the graph of a rational function $g(x)=\frac{a-bx}{x}$ the graph crosses the axis at point $A(\frac{1}{2},0)$



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i) State the domain and range of function $g(x)$

ii) find the constant a and b in $g(x)$

c) If $f(x)=3x-1$ and $g(x)=\frac{x}{3}+\frac{1}{3}$, show that f and g are inverses to each other.

7. (a) Show that the Newton Raphson formula for finding the rth root of a positive number A is given by $x_{n+1}=\frac{1}{r}\left(x_n(r-1)+\frac{A}{x_n^{\,r-1}}\right)$, hence or otherwise find the square root of 5 correct to four decimal places (take $x_0=2$ and perform 3 iterations).

(b)A root of the equation $x^2-2x-1=0$ lies between 2 and 3. Find this root using the

Secant method correct to three decimal places. Perform 3 iterations.

(c) Use the Trapezoidal and the Simpson's rules with eleven ordinates to find approximate value of $\int_0^1 \frac{dx}{3+3x^2}$ correct to four decimal places.

8. (a) Derive a formula for calculating the angle θ between two lines L_1 and L_2 which make angles θ_1 and θ_2 respectively with the x axis given that $\theta_2 > \theta_1$.

(b) Calculate the acute angle between the pair of straight lines represented bythe equation $3x^2 - 10xy + 7y^2 = 0$.

(c) Find the equations of the tangents to the circle $x^2 + y^2 - 4x - 6y + 9 = 0$ which are parallel to the line $3x + 4y = 6$.

9. (a) Integrate

(i) $\int \frac{7\cos\theta + 4\sin\theta}{2\cos\theta + 3\sin\theta} d\theta$

(ii) $\int \frac{dx}{\sqrt{5 - 4x - x^2}}$

(b) Find the area enclosed between the curve $y^2 = 4x$ and the line $y = x$

(c) A curve is given parametrically by the equations $x = \sqrt{2}(\theta + \sin\theta)$ *and* $y = \sqrt{2}(1 + \cos\theta)$.

Find the length of the curve between the points where $\theta = 0$ *and* $\theta = \frac{\pi}{2}$

10. (a). Differentiate $5x + \cos 3x$ from the first principles.

(b) If $\sin\theta = 3x^2 - 4xy + 2y^2$, show that $x\frac{\partial\theta}{\partial x} + y\frac{\partial\theta}{\partial y} = 2\tan\theta$

(c) Use Maclaurin's series to expand $\ln(1+x)$ in ascending powers of x as far the term in x^4.

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