THE UNITED REPUBLIC OF TANZANIA
PRESIDENT'S OFFICE, REGIONAL ADIMINISTRATION AND LOCAL GOVERMENT

FORM SIX SPECIAL SCHOOL JOINTS EXAMINATION

ADVANCE MATHEMATICS 1

142/1

Time: 3 Hours **Wednesday 03/March,2021 A.M**

Instructions

1. This paper consists of **Ten (10)compulsory** Questions.
2. Answer **all** questions.
3. **All** necessary workings and answers for each question done must be shown clearly.
4. Mathematical tables may be used.
5. Cellular phones and any unauthorized materials are not allowed in the examination room.
6. Write your **Number** on every page of your answer sheet(s).

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1. (a) By using a non programmable scientific calculator evaluate the following

(i) $\sqrt[3]{\dfrac{e^{-3}\text{cosec}^{-1}(3.177)}{\cos(43.7^0)\times \text{In}(87.4)}}$ put your answer to 4 decimal places

(ii) $\int_{0.1}^{0.9} \dfrac{x\sin^{-1}(x^2)}{\sqrt{1+x^2}}dx$ give your answer to 5 decimal places

(b) A population of cells doubles in size every day. After x days, the population is given by $P(x) = 60(2^x)$.

(i) What is the size of the initial population?

(ii) How many cells will there be after 12 days?

(iii) After how many days will there be two million cells?

(c) Use non programmable scientific calculator to evaluate the mean and standard deviation from the following frequency distribution.

A	9.05	12.05	15.05	18.05	21.05	24.05	27.05	30.05	33.05	36.05
F	12	30	15	39	22	28	13	19	14	10

2. (a) (i) Define sinhx and coshx in terms of exponential functions.

(ii) Using the definitions in (a) (i) above, show that $2x = In\left(\dfrac{\cosh x+\sinh x}{\cosh x-\sinh x}\right)$. Hence, by substituting $\tanh x = \dfrac{1}{3}$, deduce that $x = \dfrac{1}{2}\text{In } 2$.

(b) Solve the equations

(i) $\cosh(\text{Inx}) - \sinh\left(\text{In}\dfrac{1}{2}\text{x}\right) = 1\frac{3}{4}$

(ii) $4\sinh x + 3e^x + 3 = 0$

(c) Evaluate $\int \text{sech}\, x\, dx$

3. A manufacturer of a certain product has two stores at Kigoma and Mtwara. He wants to transport goods to two towns Singida and Moshi. He needs 60 tones and 40 tones of goods at Singida and Moshi respectively. Kigoma store contains 80 tones and 60 tones at Mtwara store. The transportation cost per tone of goods are as shown in the table below:-

TO / FROM	SINGIDA	MOSHI
KIGOMA	180,000/=	120,000/=
MTWARA	240,000/=	100,000/=

(a) How does the manufacture transport goods at a minimum cost?

(b) What is the minimum cost?

4. Below is a frequency distribution table showing the lifetimes of 400 radio tubes tested at J & L Tube company.

Life time (Hours)	300 – 399	400 – 499	500 – 599	600 – 699	700 – 799	800 – 899	900 – 999	1000 – 1099	1100 – 1199

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Number of Tubes	14	46	58	76	68	62	48	22	6

Calculate:-

(a) Mean by coding method use assumed mean as 649.5

(b) The absolute mean deviation.

(c) The variance by coding method.

(d) The semi-interquartile range.

5. (a).(i) Define subset of a set

(ii)How many subsets are in the following set ? A= {1,2,{6,7,8}}

(b). By using algebraic law of a sets simplify $A' \cap (B \cup A)$

(c) (i). Define cardinality of a set

(ii). Employees of a certain manufacturing company are categorized as follows:

A: Shift workers

B: Employees who are members of the workers union

C: Trained workers

If out of 200 shift workers, 60 are trained, 55 are trained and members of the workers union and 38 are members of the workers union but not trained. Draw the venn diagram to represent this information. How many are untrained shift workers who are not members of the union?

6. (a) The function f and g are defined as $f(x) = \begin{cases} x \text{ for } x < 0 \\ x^2 \text{ for } x \geq 0 \end{cases}$ and $g(x) = \sqrt{x}$, find the composite function $gof(x)$ and state the domain in which it is defined.

(b) Find the composite functions fog(x) and gof(x) given that $f = \{(1,6), (4,7), (5,0)\}$ and $g = \{(6,1), (7,4), (0,5)\}$

(c) Sketch the graph of $f(x) = \frac{x^3}{x^2 - 9}$.

7. (a) Derive the Newton Raphson formula for finding the solution of $f(x) = 0$,

(b) Consider the function $f(x) = 3\sin 2x - 2e^{3x} + x$. Use the Newton Raphson method starting at $x_0 = -2$ to approximate the root to 4 decimal points.

(c) Applying both Trapezium and Simpsons rule with 10 intervals to find an approximate value of the following integrals. $\int_0^2 \tan^{-1}\sqrt{x+1}\, dx$ correct to five decimal places

8. (a) L(-1, 0), M(3, 7), N(5, -2) are the mid-points of the sides BC, CA, AB respectively of the ΔABC. Find the equation of AB.

(b) A line parallel to the $x - axis$ cuts the curve $y^2 = 4x$ at point P and the line $x = -1$ at Q . Find the locus of the midpoint of the PQ.

(c) Given two circles $C_I : x^2 + y^2 = 16$ and $C_2 : x^2 + y^2 - 2x - 4y = 0$. Find

(i) The angle at which the circles C_I and C_2 intersect.

(ii) The equation of the common chord of the two circles.

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(iii) The points of intersection of the two circles.

9. (a) Integrate the following with respect to x: (a) $\int \frac{2x}{\sqrt{a+x}+\sqrt{a-x}}dx$ (b) $\int \frac{1}{1-\cos\frac{x}{2}}dx$.

(c) A curve has an equation $9y^2 = 16x^2$. Determine the length of an arc of the curve between the point $(1, 4/3)$ and the point $(4, 32/3)$.

10. (a) Use first principles to differentiate $\frac{\cos 2x}{x}$.

(b) Find $\frac{dy}{dx}$ in each of the following

(i) $y = e^{-x^3}$ (ii) $y = 7^{x^2+2x}$

(c) A cylinder of volume Vcm^3 is inscribed in a cone of radius 5cm and height of 15cm. Show that the maximum volume of the cylinder that can be inscribed in the cone is $V = \frac{500}{9}cm^3$

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