

THE UNITED REPUBLIC OF TANZANIA
REGIONAL ADMINISTRATION AND LOCAL GOVERNMENT
FORM SIX PRE – MOCK JOINT EXAMINATION
ADVANCED MATHEMATICS – 1

TIME: 3HOURS

Monday 26, October 2020

INSTRUCTIONS

1. This paper consists of **Ten (10)** questions each carrying ten (10) marks.
2. Answer **all** questions.
3. **All** work done in answering each question must be shown clearly.
4. Mathematical tables and non – programmable calculators may be used.
5. Cellular phones are strictly prohibited in examination room.
6. Write your **name** and **school name** on every page of your answer sheets.

1. By using non – programmable calculator.

(a) (i) $\sqrt[5]{\frac{(46.95)^6 \times \sin^3 57.56^\circ}{\sqrt{164.8} \times \tanh 28.6^\circ}}$ to 4 significant figures.

(ii) Compute $\frac{5^{2.3} \times \log e^{16} + e^4}{\log 24 + \sin 15.5^\circ}$ to 2 decimal places.

- (b) (i) Find the value of 't' in minutes from the formular $N = N_0 e^{kt}$, given that $N = 1 \times 10^3$, $N_0 = 2.1 \times 10^2$ and $k = 0.002$.

- (ii) The period T of oscillation on a simple pendulum was recorded according to

$T = 2\pi \sqrt{L/g}$, in which L is the length of a thread and g is an acceleration due to gravity whose value is 9.8 m/s^2 . The period was recorded under the same number of oscillations. Complete the table below to 3 decimal places. Take $\pi = \frac{22}{7}$.

Length (cm)	Period (T) in sec.
26	
35	

2. (a) Show that

(i) $\frac{1 + \tanh x}{1 - \tanh x} = e^{2x}$

(ii) $\coth^{-1} x = \frac{1}{2} \ln \left(\frac{x+1}{x-1} \right)$.

- (b). Solve $2 \cosh 2x + 10 \sinh x = -2$.

(c) Sketch $y = \cosh x$ and $y = \cosh^{-1}x$ on the same xy plane and determine its domain.

3. (a) A catering agency has two kitchens to prepare food at two places A and B. From these places “Mild – day meal” is to be supplied to three different schools situated at P, Q and R. The monthly requirements of the schools are respectively 40, 40 and 50 food packets. A packet contains lunch for 1000 students. Preparing capacity of kitchen A and B are 60 and 70 packets per month respectively. The transportation cost per packet for the kitchen to the schools is given below.

TRANSPORTATION COST PER PACKET (IN &)		
TO	FROM	
	A	B
P	5	4
Q	4	2
R	3	5

How many packets from each kitchen should be transported school so that the cost of transportation is minimum. Also find the minimum cost.

(b). A village in Zimbabwe has 175 acres of form land for breeding pigs and sheep. The average 0.5 acre of land is required to support either a sheep or a pig. A village can produce up to 7000 man hours of labour for breeding. It takes 15 man hours of labour to breed a pig and 20 man hours of labour to breed a sheep. Although the village is willing to breed atleast 100pgs, it doesnot wish to breed more than 200 sheep at a time. Also breeding of pigs is limited to 250. It is expected that each pig will contribute 300 Zimbabwe dollars, while each sheep will contribute 350 Zimbabwe dollars Formulate this as linear programming problem.

4. (a) i) Calculate the mean and standard deviation of the four numbers 2, 3, 6 and 9.
ii) Two numbers a and b are to be added to this set four numbers in a i) above such that the mean is increased by 1 and the variance is increased 2.5. Find the numbers a and b .

(b). i) Show that $\frac{\sum f(x_i - \bar{x}_i)}{\sum f} = 0$ as applied in statistics.

ii) A teacher at Milambo High School decided to record weights of his 200 students, as shown in frequency distribution below:-

Weight (kg)	24 – 29	30 – 35	36 – 41	42 – 47	48 – 53
Number of students	22	45	72	43	18

Find the mean deviation.

5. (a) i) Let $A = \{a, b\}$, $B = \{2, 4, 6\}$ and $C = \{1, 2, 3, 4, 5, 6\}$ prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

ii) The formula $A - B = A \cap B'$ defines the difference operation in terms of the operations of intersection and complement. Find a formula that defines the union of two sets $A \cup B$ in terms of the operation of intersection and complement.

iii) find the powerset of S , $P(S)$, of the set $S = \{3, 1, 4\}$

(b). 72 students at Tabora School participate in football and volleyball games. Those who participate in football and volleyball are 20% of those who participate in football only while those who participate in volleyball only are five times those who participate in football. Use Venn diagram to find the number of students who participate in.

- i. Volleyball
- ii. Both games.

6. (a) If the function f and g are two functions that defined as $f(x) = \ln \sqrt{\frac{l+x^2}{l-x^2}}$ and

$g(x) = e^{2x}$. Find the value of $\text{fog} \left(\log_e \frac{1}{\sqrt{2}} \right)$.

(b) A function $f(x)$ is defined as $f(x) = \frac{x^2}{x^2 - x - 2}$, determine.

- i) Vertical and horizontal asymptotes.
- ii) The turning points of the function.
- iii) Sketch the graph of $f(x)$ and state its domain.
- iv) State the value of $f(x)$ for which have no real value of x .

7. (a) i) By using Newton's Raphson Method, write down the iterative formula for extracting the square root of the square root of the number A .

ii) Determine the reciprocal of 3, starting with 0.5. Carry out four iterations.

(b). Estimate to 4 decimal places, $\int_0^1 \frac{1}{l+x^2} dx$, using five ordinates by using

- i. The trapezium rule.
- ii. Simpson's rule.

Hence determine the accuracy of your estimated values. (Take $\pi = 3.1416$)

8. (a) i) Show that $M(s, t) = \left(\frac{as_2 - bs}{a-b}, \frac{at_2 - bt_1}{a-b} \right)$ where (s, t) is the point which divides the line segment in the ratio $a:b$ and $a \neq b$.

ii) Find the two points of trisection of the line joining the point $A(2, 4)$ and $B(3, 6)$.

(b). Without using Pythagoras theorem and finding the angles, show that $(4, 4)$, $(3, 5)$ and $(-1, -1)$ are the vertices of a right angled triangle.

(c) Show that circles $x^2 + y^2 + 4x + y - 3 = 0$, $x^2 + y^2 - x - y - 1 = 0$ and $x^2 + y^2 + 14x + 5y - 7 = 0$ are co-axial.

9. (a) Evaluate $\int \frac{(x^2+1)}{(x+1)^2} dx$.
- (b) Show that $\int_0^{\frac{\pi}{2}} \frac{a \sin x + b \cos x}{\sin x + \cos x} dx = \frac{\pi}{4} (a + b)$
- (c) Find the area of a curve between the x – axis and the first period of the cycloid $x = t - \sin t, y = 1 - \cos t$. The value of t run from 0 and 2π
10. (a) If $\sqrt{x^2 + y^2} = ae^{\tan^{-1}(\frac{y}{x})}$, show that $\frac{dy}{dx} = \frac{x+y}{x-y}$.
- (b) If $x = a \sin 2t(1 + \cos 2t)$ and $y = b \cos 2t(1 - \cos 2t)$.
Evaluate $\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}}$ in its simplest form.
- (c). A circular metal plate expands under heating so that its radius increases by 2%. Find the approximate increase in the area of the plate if the radius of the plate before heating is 10cm.