

ADVANCED MATH'S 2

FORM SIX-JOINT

MARKING SCHEME

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1 a) i) Given; $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$

Solution

$P(B) = y_1$ For Mutually exclusive

From $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ——— 0.5 Marks

But $P(A \cap B) = 0$ For Mutually exclusive ——— 0.5 Marks

$$P(A \cup B) = P(A) + P(B)$$

$$\frac{3}{5} = \frac{1}{2} + y_1$$

$$y_1 = \frac{1}{10}$$
 ——— 0.5 Marks

But $P(B) = y_2$ For Independent event

From $P(A \cap B) = P(A) \times P(B)$ ——— 0.5 Marks

But $P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

$$\frac{3}{5} = \frac{1}{2} + y_2 - \frac{y_2}{2}$$
 ——— 0.5 Marks

$$\frac{1}{10} = \frac{y_2}{2}$$

$$y_2 = \frac{1}{5}$$

$$y_1 + y_2 = \frac{1}{10} + \frac{1}{5} = \frac{3}{10}$$

$\therefore$ Value of $y_1 + y_2 = \frac{3}{10}$ ——— 0.5 Marks

ii) Given; Box with correction Pen = {1, 2, 3, 4, 5, 6, 7, 8 and 9}

Coin = {H, T} and Die = {1, 2, 3, 4, 5, 6}

Even number = {2, 4, 6, 8} From the Box

Prime number = {2, 3, 5, 7} From the Box

= Break the experiment into two different and work out the number of outcomes separately. Then add these results

First Experiment

Second experiment

[Even number) and (toss a coin)] or [Prime number) and (throw a die)] ——— 0.5 Marks

$$(4 \times 2) +$$

$$(4 \times 6)$$

$$8 + 24 = 32$$

$\therefore$ 32 outcomes are possible ——— 0.5 Marks

1 (b) i) Father and Mother next to each other

$$= (n-1-1)! \times 2! \quad \text{--- 01 Marks}$$

$$= (n-2)! \times 2! \quad \text{But } n=6$$

$$= (6-2)! \times 2!$$

$$= 4! \times 2!$$

$$= 48 \text{ ways}$$

$$\therefore \text{Number of ways} = 48 \text{ ways} \quad \text{--- 01 Marks}$$

ii) Father and Mother not next to each other

$$\text{Not next} = \text{Total} - \text{Next} \quad \text{--- 0 1/2 Marks}$$

$$\text{Total} = (n-1)! = (6-1)! = 5! \quad \text{--- 0 1/2 Marks}$$

$$\text{Total} = 120 \text{ ways}$$

$$\text{Not next} = 120 - 48$$

$$= 72 \text{ ways}$$

$$\therefore \text{Number of ways} = 72 \text{ ways} \quad \text{--- 01 Marks}$$

c) Given, $p = 0.0001$, $n = 2000$, $\lambda = np$
 $\lambda = 2000(0.0001) = 2$

$$\text{At least 2 will suffer} = \{x = 2, 3, 4, \dots, 2000\} = (x \geq 2)$$

$$P(x \geq 2) = 1 - P(x \leq 1)$$

$$P(x \geq 2) = 1 - P(x = 1 \text{ or } 0)$$

$$P(x \geq 2) = 1 - [P(x=1) + P(x=0)]$$

$$\text{But } P(x=x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad \text{--- 01 Marks}$$

$$P(x \geq 2) = 1 - \left[\frac{\lambda^1 e^{-\lambda}}{1!} + \frac{\lambda^0 e^{-\lambda}}{0!} \right] \quad \text{--- 01 Marks}$$

$$= 1 - [2e^{-2} + e^{-2}]$$

$$= 1 - 3e^{-2}$$

$$P(x \geq 2) = 0.5940$$

$$\therefore \text{The probability is } 0.5940 \quad \text{--- 01 Marks}$$

$$2b)(ii) (p \wedge q) \vee [(\sim r \wedge (q \wedge p))] \dots \text{Given.}$$

$$(p \wedge q) \vee [(\sim r \wedge (p \wedge q))] \dots \text{Commutative law.}$$

$$((p \wedge q) \wedge T) \vee [(\sim r \wedge (p \wedge q))] \dots \text{Identity law.}$$

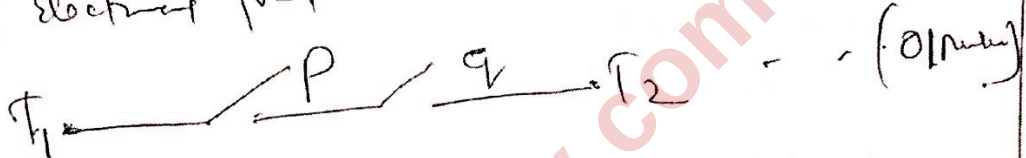
$$((p \wedge q) \wedge T) \vee [(p \wedge q) \wedge \sim r] \dots \text{Contradiction law.} \quad \text{--- (0/1 mark)}$$

$$(p \wedge q) \wedge [T \vee \sim r] \dots \text{Distributive law.}$$

$$(p \wedge q) \wedge T \dots \text{Identity law.} \quad \text{--- (0/1 mark)}$$

$$p \wedge q \dots \text{Identity law.} \quad \text{--- (0/1 mark)}$$

The electric network.



C(i) Required to simplify the statement T.

Select the "F" in the T column.

$$(\sim p \vee q \vee \sim M) \wedge (\sim p \vee \sim q \vee M) \wedge (\sim p \vee \sim q \vee \sim M) \dots \text{Given.}$$

$$(\sim p \vee q \vee \sim M) \wedge [(\sim p \vee \sim q) \vee (M \wedge \sim M)] \dots \text{Distributive law.}$$

$$(\sim p \vee q \vee \sim M) \wedge [(\sim p \vee \sim q) \vee F] \dots \text{Contradiction law.}$$

$$(\sim p \vee q \vee \sim M) \wedge (\sim p \vee \sim q) \dots \text{Identity law.} \quad \text{--- (0/1 mark)}$$

$$[\sim p \vee (q \vee \sim M)] \wedge (\sim p \vee \sim q) \dots \text{Identity law.}$$

$$\sim p \vee [(q \vee \sim M) \wedge (\sim q)] \dots \text{Distributive law.}$$

(0/1 mark)

∴ 80.23% of students scored below Tom's score.
Therefore he scored, Tom is better than 70% of the
students who took the test. Tom will be admitted
to university. (10½ marks)

2 (a) (i) If Contrapositive you can comprehend geometry then -
you know how to reason deductively. . . . (10½ marks)

(ii) If Converse you cannot comprehend geometry then you
do not know how to reason deductively. . . . (10½ marks)

(ii) Contrapositive

If $a - b$ is not always a positive integer, then
two integers a and b are such that a is not q
times b (10½ marks)

Converse

If $a - b$ is always a positive integer, then two
integers a and b are such that a is not greater
than b (10½ marks)

(iii) Contrapositive

If a quadrilateral does not have two pairs of
parallel sides, then it is not a rectangle. . . . (10½ marks)

Converse

If a quadrilateral has two pairs of parallel sides,
then it is a rectangle. . . . (10½ marks)

(03)

2

(b)(i) let p : I work hard q : I get raise r : I get a new job.

$$((p \rightarrow q) \wedge (q \rightarrow \sim r) \wedge r) \rightarrow \sim p \quad \text{--- (001) true}$$

Then table of $((p \rightarrow q) \wedge (q \rightarrow \sim r) \wedge r) \rightarrow \sim p$

					A	B	C	
p	q	r	$\sim p$	$\sim r$	$p \rightarrow q$	$q \rightarrow \sim r$	$A \wedge B \wedge r$	Con.
T	T	T	F	F	T	F	F	T
T	T	F	F	T	T	T	F	T
T	F	T	F	F	F	T	F	T
T	F	F	F	T	F	T	F	T
F	T	T	T	F	T	F	F	T
F	T	F	T	T	T	T	F	T
F	F	T	T	F	T	T	T	T
F	F	F	T	T	T	T	F	T

Since the final column has all T, therefore, the Argument is valid (001) true.

(25)

2 (i) ~~$\sim P \vee [(G \vee \sim M) \wedge \sim G]$~~ ... Distribution law.

2 (c)(i) Required to simplify the statement.
Select the 'T' in the column.

$(P \wedge G \wedge M) \vee (P \wedge G \wedge \sim M) \vee (P \wedge \sim G \wedge \sim M) \vee (\sim P \wedge G \wedge M)$... Given.

$[P \wedge G] \wedge (M \vee \sim M) \vee [(P \wedge \sim G) \wedge M] \vee (P \wedge \sim G \wedge \sim M) \vee [\sim P \wedge G \wedge M]$... Given.

$(P \wedge G) \wedge (M \vee \sim M) \vee (P \wedge \sim G) \wedge (M \vee \sim M) \vee (\sim P \wedge G \wedge M)$... Distribution law. 0/1

$[(P \wedge G) \wedge T] \vee [(P \wedge \sim G) \wedge T] \vee (\sim P \wedge G \wedge M)$... Complementing law.

$(P \wedge G) \vee (P \wedge \sim G) \vee (\sim P \wedge G \wedge M)$... Identity law.

$P \wedge (G \vee \sim G) \vee (\sim P \wedge G \wedge M)$... Distribution law. 0/1

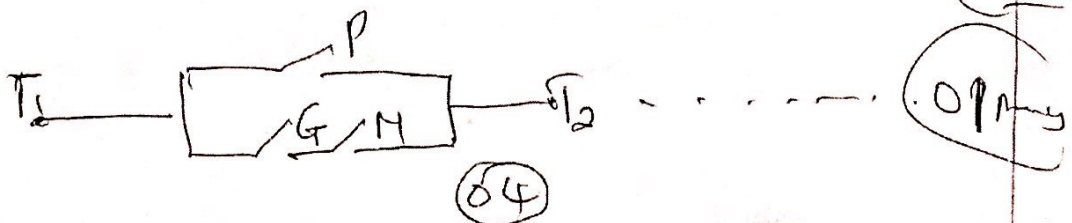
$P \wedge T \vee (\sim P \wedge G \wedge M)$... Complementing law.

$P \vee (\sim P \wedge G \wedge M)$... Identity law. 0/1

$(P \vee \sim P) \wedge (P \vee (G \wedge M))$... De Morgan law.

$T \wedge (P \vee (G \wedge M))$... Complementing law.

$P \vee (G \wedge M)$... Identity law. 0/1



3 (i) $r_m = 5 \text{ unit}$

$$\vec{r} = 2\hat{i} - 2\hat{j} + k$$

$$|\vec{r}| = \sqrt{2^2 + (-2)^2 + 1^2} = \sqrt{9} = 3 \dots \text{ (001/2/100)}$$

$$F = r_m \left(\frac{\vec{r}}{|\vec{r}|} \right) = 5 \left(\frac{2\hat{i} - 2\hat{j} + k}{3} \right) = \frac{5}{3} (2\hat{i} - 2\hat{j} + k) \dots \text{ (001/2/100)}$$

$$F = \frac{5}{3} (2\hat{i} - 2\hat{j} + k) \dots \text{ (01/100)}$$

No distance from $(5, 3, 7)$ to $(1, 2, 3)$

$$d = (5, 3, 7) - (1, 2, 3) = (4, 1, 4)$$

$$d = (4, 1, 4) \dots \text{ (11)}$$

from, work done = $F \cdot d$

$$\text{Work done} = \frac{5}{3} \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 1 \\ 4 \end{pmatrix} = \frac{5}{3} (8 - 2 + 4) = \frac{50}{3}$$

$$\text{Work done} = \frac{50}{3} \text{ joules} \dots \text{ (01)}$$

(ii) $\phi(x, y, z) = x^2 y z$

$$\vec{a} = 3x^2 y \hat{i} + y z^2 \hat{j} - x z \hat{k}$$

$$(\phi_a) = x^2 y z (3x^2 y \hat{i} + y z^2 \hat{j} - x z \hat{k}) \dots \text{ (01/100)}$$

$$(\phi_a) = 3x^4 y^2 z \hat{i} + x^2 y^2 z^3 \hat{j} - x^3 y z^2 \hat{k} \dots \text{ (01/100)}$$

$$\frac{\partial}{\partial z} (\phi_a) = 3x^4 y^2 \hat{i} + 3x^2 y^2 z^2 \hat{j} - 2x^3 y z \hat{k}$$

$$\frac{\partial^2}{\partial z^2} (\phi_a) = 6x^4 y \hat{i} + 6x^2 y z \hat{j} - 2x^3 \hat{k} \dots \text{ (01/100)}$$

(06)

at point $(1, -2, -1)$

$$\frac{\partial^2}{\partial y \partial z} (\phi_a) = 6(1)^4(-2)^2 + 6(1)^2(-2)^2(-1)^2 - 2(1)^3(-1)^2$$

$$\frac{\partial^2}{\partial y \partial z} (\phi_a) = 12\hat{i} + 12\hat{j} + 2\hat{k} \quad \text{--- OI Mark}$$

$$\frac{\partial}{\partial x} \frac{\partial^2}{\partial y \partial z} (\phi_a) = -12\hat{i} - 12\hat{j} + 2\hat{k} \quad \text{--- OI Mark}$$

(b) ~~Q~~ from $(a+b)^2 = a^2 + 2ab + b^2$

we know $a^2 = |a|^2$, $b^2 = |b|^2$ --- OI Mark

$$a \cdot b = |a||b|\cos\theta$$

$$(a+b)^2 = |a|^2 + 2|a||b|\cos\theta + |b|^2$$

$$\text{but } |a| = |b| = 1$$

$$(a+b)^2 = 1 + 2\cos\theta + 1 \quad \text{--- OI Mark}$$

$$(a+b)^2 = 2 + 2\cos\theta$$

$$\frac{1}{2}(a+b)^2 = 1 + \cos\theta$$

$$\frac{1}{2}(a+b)^2 = 2\cos^2\frac{\theta}{2}$$

$$\frac{1}{4}(a+b)^2 = \cos^2\frac{\theta}{2}$$

$$\cos(\frac{\theta}{2}) = \frac{1}{2}(a+b) \quad \text{--- OI Mark}$$

$$\therefore \frac{1}{2}(a+b) = \cos(\frac{\theta}{2}) \quad \text{Hence Proved. --- OI Mark}$$

Q7.

~~$x^2 + 2x + 1 = 0$~~
 ~~$x = -1$~~

4 (Q) $x^2 + (-2\cos\theta)x + 1 = 0$ Compare with
 $ax^2 + bx + c = 0$

$a = 1, b = -2\cos\theta, c = 1$

$$x = \frac{2\cos\theta \pm \sqrt{(-2\cos\theta)^2 - 4 \times 1 \times 1}}{2 \times 1} \quad \leftarrow \text{0/1 mark}$$

$$x = \frac{2\cos\theta \pm 2\sqrt{-(\sin^2\theta)}}{2}$$

$$x = \frac{2\cos\theta \pm 2i\sin\theta}{2} \quad \leftarrow \text{0/1 mark}$$

$$x = \cos\theta \pm i\sin\theta$$

$\therefore x = \cos\theta + i\sin\theta$ or $x = \cos\theta - i\sin\theta$

0/1 mark

(09)

$$4 (b) \text{ Given } z = \sin \frac{\pi}{5} + i(1 - \cos \frac{\pi}{5})$$

$$| \text{Mod}(z) | = \sqrt{\sin^2 \frac{\pi}{5} + (1 - \cos \frac{\pi}{5})^2} \quad \text{--- 0/1 mark}$$

$$| \text{Mod}(z) | = \sqrt{\sin^2 \frac{\pi}{5} + \cos^2 \frac{\pi}{5} + 1 - 2 \cos \frac{\pi}{5}}$$

$$| \text{Mod}(z) | = \sqrt{1 + 1 - 2 \cos \frac{\pi}{5}} \quad \text{--- 0/1 mark}$$

$$| \text{Mod}(z) | = \sqrt{2(1 - \cos \frac{\pi}{5})}$$

$$| \text{Mod}(z) | = \sqrt{2(2 \sin^2 \frac{\pi}{10})}$$

$$| \text{Mod}(z) | = 2 \sin \frac{\pi}{10} \quad \text{--- 0/1 mark}$$

$$\text{Arg}(z) = \tan^{-1} \left(\frac{1 - \cos \frac{\pi}{5}}{\sin \frac{\pi}{5}} \right)$$

$$\text{Arg}(z) = \tan^{-1} \left(\frac{2 \sin^2 \frac{\pi}{10}}{2 \sin \frac{\pi}{10} \cos \frac{\pi}{10}} \right) \quad \text{--- 0/1 mark}$$

$$\text{Arg}(z) = \tan^{-1}(\tan \frac{\pi}{10})$$

$$\text{Arg}(z) = \frac{\pi}{10}$$

$$\therefore \text{Arg}(z) = \frac{\pi}{10} \quad \text{--- 0/1 mark}$$

(10)

$$4 \quad (c) \quad \frac{(3+4i)(1+2i)}{1+3i} = q(1+i)$$

$$\frac{3(1+2i) + (4i)(1+2i)}{1+3i} = q(1+i) \quad \text{--- (O/P)} \quad \text{---}$$

$$\frac{3+10i-8}{1+3i} = q+qi$$

$$\frac{-5+10i}{(1+3i)(1-i)} = q+qi$$

$$\frac{-5(1-i) + (10i)(1-i)}{10} = q+qi \quad \text{--- (O/P)} \quad \text{---}$$

$$-5+5i+10i+10 = 10(q+qi)$$

$$25+25i = 10q+10qi$$

$$10q = 25$$

$$q = \frac{25}{10} = \frac{5}{2}$$

$$\text{From} \quad \frac{(3+4i)(1+2i)}{1+3i} = \frac{5}{2}(1+i) \quad \text{--- (O/P)} \quad \text{---}$$

Compare argument both sides

$$\arg\left(\frac{(3+4i)(1+2i)}{(1+3i)}\right) = \arg\left(\frac{5}{2}(1+i)\right)$$

$$\arg(3+4i) + \arg(1+2i) - \arg(1+3i) = \arg\left(\frac{5}{2}(1+i)\right)$$

$$\tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}(2) - \tan^{-1}(3) = \tan^{-1}\left(\frac{5}{2}\right)$$

$$\tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}(2) - \tan^{-1}(3) = \frac{\pi}{4} \quad \text{--- (O/P)} \quad \text{---}$$

(11)

(d)

$$z = x + iy$$

$$\frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1} = \frac{(2x+1)+2yi}{(-y+1)+ix}$$

$$\frac{((2x+1)+2yi)((-y+1)-ix)}{((-y+1)+ix)((-y+1)-ix)}$$

$$\frac{(2x+1)(-y+1) - ix(2x+1) + 2yi(-y+1) - 2i^2xy}{(-y+1)^2 + x^2}$$

$$\frac{(2x+1)(-y+1) + 2xy + i((-2x^2+x) + 2y(-y))}{(-y+1)^2 + x^2}$$

$$\frac{(2x+1)(-y+1) - i(2x^2+x) + i(-2y^2+2y) + 2xy}{(-y+1)^2 + x^2}$$

$$\frac{(2x+1)(-y+1) + 2xy + i(2y - 2y^2 - 2x^2 - x)}{(-y+1)^2 + x^2}$$

$$\text{Im} \left(\frac{2z+1}{iz+1} \right) = -2$$

$$\frac{2y - 2y^2 - 2x^2 - x}{(-y+1)^2 + x^2} = -2$$

$$2y - 2y^2 - 2x^2 - x = -2x^2 - 2y^2 + 4y - 2$$

$$2y - x = 4y - 2$$

$$x + 2y - 2 = 0$$

$$y = -\frac{1}{2}x + 1$$

∴ It is a straight line with slope of $-\frac{1}{2}$.

(12)

SECTION B

5. (a) Let $y = \alpha + \beta \Rightarrow \tan(\alpha + \beta) = \tan y$.

$$y = \tan^{-1} \left(\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \right) \quad \text{--- 01 mark}$$

$$y = \tan^{-1} \left(\frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \left(\frac{m}{m+1}\right)\left(\frac{1}{2m+1}\right)} \right)$$

$$y = \left(\frac{\frac{m(2m+1) + (m+1)}{(m+1)(2m+1)}}{1 - \frac{m}{(m+1)(2m+1)}} \right) \quad \text{--- 01 mark}$$

$$y = \tan^{-1} \left(\frac{\frac{m(2m+1) + (m+1)}{(m+1)(2m+1)}}{\frac{(m+1)(2m+1) - m}{(m+1)(2m+1)}} \right) \quad \text{--- 01 mark}$$

$$y = \tan^{-1} \left(\frac{m(2m+1) + m+1}{(m+1)(2m+1) - m} \right)$$

$$y = \tan^{-1} \left(\frac{2m^2 + 2m + 1}{2m^2 + 2m + 1} \right)$$

$$y = \tan^{-1}(1) = \frac{\pi}{4} \quad \text{--- 01 mark}$$

$$\alpha + \beta = \frac{\pi}{4}.$$

(13)

$$5) (b). 24 \cos\left(\frac{\pi}{3}x\right) + 10 \sin\left(\frac{\pi}{3}x\right) = 13.$$

Consider:

$$24 \cos\left(\frac{\pi}{3}x\right) + 10 \sin\left(\frac{\pi}{3}x\right) = R \cos\left(\frac{\pi}{3}x - \alpha\right)$$

$$24 \cos \frac{\pi}{3}x + 10 \sin \frac{\pi}{3}x = R \cos \frac{\pi}{3}x \cos \alpha + R \sin \frac{\pi}{3}x \sin \alpha$$

— 01 rule

By Comparing.

$$10 \sin\left(\frac{\pi}{3}x\right) = R \sin \alpha \sin \frac{\pi}{3}x.$$

$$R \sin \alpha = 10 \quad \text{--- (i)}$$

$$R \cos \alpha = 24 \quad \text{--- (ii)}$$

$$R^2 (\sin^2 \alpha + \cos^2 \alpha) = 24^2 + 10^2$$

$$R = \sqrt{676} = 26.$$

$$\tan \alpha = \frac{10}{24} \Rightarrow \alpha = \tan^{-1}\left(\frac{10}{24}\right)$$

— 01 rule

$$\alpha = \tan^{-1}\left(\frac{5}{12}\right)$$

$$26 \cos\left(\frac{\pi}{3}x - \tan^{-1}\left(\frac{5}{12}\right)\right) = 13.$$

$$\cos\left(\frac{\pi}{3}x - \tan^{-1}\left(\frac{5}{12}\right)\right) = \frac{1}{2}.$$

— 01 rule

$$\frac{\pi}{3}x - \tan^{-1}\left(\frac{5}{12}\right) = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\frac{\pi}{3}x - \tan^{-1}\left(\frac{5}{12}\right) = 2\pi n \pm \frac{\pi}{3}$$

$$x = \frac{6\pi n \pm 1 + \frac{3}{\pi} \tan^{-1}\left(\frac{5}{12}\right)}{\pi}$$

— 01 rule

$$\therefore x = (6n \pm 1) + \frac{3}{\pi} \tan^{-1}\left(\frac{5}{12}\right).$$

14.

5) (c) (i) Given that $\frac{1}{2\cos\theta + 5\sin\theta + 7}$.

Let $2\cos\theta + 5\sin\theta = R\cos(\theta - \alpha)$ --- 01 mark

$$2\cos\theta + 5\sin\theta = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$$

$$R\cos\alpha = 2 \quad \text{--- (i)}$$

$$R\sin\alpha = 5 \quad \text{--- (ii)} \quad \text{--- 01 mark}$$

$$R^2\cos^2\alpha + R^2\sin^2\alpha = 5^2 + 2^2$$

$$R^2 = 29$$

$$R = \sqrt{29}, \quad \alpha = \tan^{-1}\left(\frac{5}{2}\right) = \dots$$

$$\sqrt{29} \cos(\theta - 21.1^\circ)$$

Max. value $\sqrt{29} \cos(\theta - 21.1^\circ)$

$$\cos(\theta - 21.1^\circ) = 1$$

$$\therefore \text{max. value} = \sqrt{29} \quad \text{--- 01 mark}$$

Min. value $\sqrt{29} \cos(\theta - 21.1^\circ)$

$$\cos(\theta - 21.1^\circ) = -1$$

$$\therefore \text{Min. value} = -\sqrt{29}$$

$$\frac{1}{\sqrt{29} + 7} = 0.0807 \quad \text{--- 01 mark}$$

$$\frac{1}{-\sqrt{29} + 7} = 1.6148$$

$\therefore$ The max. value is 1.6148 and min. value is 0.0807. --- 01 mark

(15)

5 (vii) Prove the L.H.S.

$$\frac{\tan x + \sec x + 1}{\tan x - \sec x + 1} = \frac{\sin x + 1 - \cos x}{\sin x - 1 + \cos x}$$

L.H.S.

$$\frac{\sin x + 1 - \cos x}{\sin x - 1 + \cos x} = \frac{2 \sin \frac{x}{2} \cos \frac{x}{2} + 1 - (1 - 2 \sin^2 \frac{x}{2})}{2 \sin \frac{x}{2} \cos \frac{x}{2} - 1 + (1 - 2 \sin^2 \frac{x}{2})}$$

$$= \frac{\cos \frac{x}{2} + \sin \frac{x}{2}}{\cos \frac{x}{2} - \sin \frac{x}{2}} \quad \left\{ \begin{array}{l} \text{Rationalize the} \\ \text{denominator} \end{array} \right.$$

$$\frac{\cos \frac{x}{2} + \sin \frac{x}{2}}{\cos \frac{x}{2} - \sin \frac{x}{2}} \cdot \frac{(\cos \frac{x}{2} + \sin \frac{x}{2})}{(\cos \frac{x}{2} + \sin \frac{x}{2})} \quad \dots \quad \text{O.H.S.}$$

$$\frac{1 + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}$$

$$\frac{1 + \sin x}{\cos x}$$

∴ Here prove that $\frac{\tan x + \sec x + 1}{\tan x - \sec x + 1} = \frac{1 + \sin x}{\cos x}$

--- O.H.S.

17

101

(d) $z = x + iy$

$$\frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1} = \frac{(2x+1)+2yi}{(-y+1)+ix}$$

$$\frac{(2x+1)+2yi}{(-y+1)+ix} \cdot \frac{(-y+1)-ix}{(-y+1)-ix}$$

$$\frac{(2x+1)(-y+1) - i(2x+1) + 2yi(-y+1) - 2i^2xy}{(-y+1)^2 + x^2}$$

$$\frac{(2x+1)(-y+1) + 2xy + i(-2x^2+x) + 2yi(-y+1) - 2xy}{(-y+1)^2 + x^2}$$

$$\frac{(2x+1)(-y+1) - i(2x^2+x) + i(-2y^2+2y) + 2xy}{(-y+1)^2 + x^2}$$

$$\frac{(2x+1)(-y+1) + 2xy}{(-y+1)^2 + x^2} + i \left(\frac{2y - 2y^2 - 2x^2 - x}{(-y+1)^2 + x^2} \right)$$

Im $\left(\frac{2z+1}{iz+1} \right) = -2$

$$\frac{2y - 2y^2 - 2x^2 - x}{(-y+1)^2 + x^2} = -2$$

$$2y - 2y^2 - 2x^2 - x = -2x^2 - 2y^2 + 4y - 2$$

$$2y - x = 4y - 2$$

$$x + 2y - 2 = 0$$

$$y = -\frac{1}{2}x + 1$$

∴ It is a straight line with slope of $-\frac{1}{2}$.

(12)

Q6 (i) Given $a^x = \left(\frac{a}{k}\right)^y$ { apply ln both side }

$$x \ln a = y \ln \left(\frac{a}{k}\right)$$

$$x \ln a = y \ln a - y \ln k.$$

$$(x-y) \ln a = -y \ln k \quad \dots (i)$$

Also given $a^x = k^m$.

$$x \ln a = m \ln k.$$

$$\frac{x \ln a}{m} = \ln k \quad \dots (ii)$$

Substitute eqn (ii) into eqn (i)

$$(x-y) \ln a = -y \left(\frac{x \ln a}{m} \right)$$

$$x-y = -\frac{y x}{m}$$

$$mx - my = -xy$$

$$y = \frac{mx}{m-x}$$

Hence shown that $y = \frac{mx}{m-x}$

Q(1) Given $\frac{8x^2 - x - 1}{(x-1)^2}$

--- (i) ---

Let $x-1 = u$, $x = u+1$ --- (ii)

Substitute eqn (ii) into eqn (i)

$$\frac{8(u+1)^2 - (u+1) - 1}{u^2}$$

$$\frac{8u^2 + 16u + 6}{u^2} = 8 + \frac{16}{u} + \frac{6}{u^2}$$

(16)

Q (b)(ii) $2 \times 3 + 3 \times 4 + 4 \times 5 + \dots$

Separate the terms of the above series and terminate the general term.

$$2 + 3 + 4 + 5 + \dots + (n+1)$$

$$3 + 4 + 5 + 6 + \dots + (n+2)$$

Combine the above series.

$$2 \times 3 + 3 \times 4 + 4 \times 5 + \dots + (n+1)(n+2)$$

Represent a series in Sigma notation.

$$2 \times 3 + 3 \times 4 + 4 \times 5 + \dots + (n+1)(n+2) = \sum_{r=1}^n (r+1)(r+2)$$

$$\sum_{r=1}^n (r+1)(r+2) = \sum_{r=1}^n (r^2 + 3r + 2) = \sum_{r=1}^n r^2 + 3 \sum_{r=1}^n r + \sum_{r=1}^n 2$$

$$= \frac{n}{6} (n+1)(2n+1) + \frac{3}{2} n(n+1) + 2n$$

$$= \frac{n}{6} (2n^2 + 12n + 22) = \frac{n}{3} (n^2 + 6n + 11)$$

$$\therefore \sum_{r=1}^n (r+1)(r+2) = \frac{n}{3} (n^2 + 6n + 11)$$

(18)

(10) Given the proposition.

(6)(i) $2^{n+2} + 3^{2n+1} = 7M$, where M is positive integer

Step 1, $n=1$, $2^{1+2} + 3^{2 \cdot 1 + 1} = 7M$... (i)

$35 = 7M$, it is true for $n=1$... (i)

Assume $n=2$, $2^{2+2} + 3^{2(2)+1}$

$259 = 7M$... it is true for $n=2$... (ii)

Step 2: Assume it is true for $n=k$... (iii)

$2^{k+2} + 3^{2k+1} = 7M$,

$2^k = \frac{7M - 3^{2k+1}}{4}$

Step 3: We need to prove that it is true for $n=k+1$

$2^{k+3} + 3^{2k+3} = 2^3(2^k) + 3^3(3^{2k})$... (iv)

$2^k = \frac{7M - 3^{2k+1}}{4}$ for eqn (iii)

$2^{k+3} + 3^{2k+3} = 2(7M - 3^{2k+1}) + 27(3^{2k})$... (v)

$2^{k+3} + 3^{2k+3} = 14M - 6(3^{2k}) + 27(3^{2k})$... (vi)

$2^{k+3} + 3^{2k+3} = 14M + 21(3^{2k})$... (vii)

$= 7(2M + 3(3^{2k}))$

$\therefore 2^{n+2} + 3^{2n+1}$ is divisible by 7. $\therefore$ Proved

(ii) Soln. Define the General term of a series.

$2+3+4+5+\dots+(n+1) < 3+4+5+\dots+(n+2)$
Divide the General term to the given series.

$$\frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \dots + \frac{1}{(n+1)(n+2)} \quad - \text{O/M}$$

Express in Sigma notation.

$$\frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \dots + \frac{1}{(n+1)(n+2)} = \sum_{r=1}^n \frac{1}{(r+1)(r+2)}$$

$$\text{Partial } \frac{1}{(r+1)(r+2)} = \frac{A}{r+1} + \frac{B}{r+2} \quad - \text{O/M}$$

$$1 = A(r+2) + B(r+1)$$

Equating coefficients of like terms

$$2A + B = 1 \quad \text{--- (i)}$$

$$A + B = 0 \quad \text{--- (ii)}$$

Solve the further eqn.

$$A = 1 \text{ and } B = -1$$

$$\text{Thus } \frac{1}{(r+1)(r+2)} = \frac{1}{r+1} - \frac{1}{r+2}$$

$$\sum_{r=1}^n \frac{1}{(r+1)(r+2)} = \sum_{r=1}^n \left(\frac{1}{r+1} - \frac{1}{r+2} \right)$$

$$\text{When } r=1, \quad \frac{1}{2} - \frac{1}{3}$$

$$r=2, \quad \frac{1}{3} - \frac{1}{4}$$

$$r=3, \quad \frac{1}{4} - \frac{1}{5}$$

$$\vdots$$

$$r=n-2, \quad \frac{1}{n-1} - \frac{1}{n}$$

$$r=n-1, \quad \frac{1}{n} - \frac{1}{n+1}$$

$$r=n, \quad \frac{1}{n+1} - \frac{1}{n+2}$$

(19)

$$\frac{1}{(r+1)(r+2)} = \frac{1}{2} \left(\frac{1}{r+1} - \frac{1}{r+2} \right)$$

$$\frac{1}{(r+1)(r+2)} = \frac{1}{2} \left(\frac{1}{r+1} - \frac{1}{r+2} \right)$$

7 (Q) Form the D.E. is

$$y = (A \cos x + B \sin x) e^{-x}$$

multiply by e^x both side = - - - 0/m

$$y e^x = A \cos x + B \sin x$$

$$e^x y + e^x \frac{dy}{dx} = -A \sin x + B \cos x \quad - - - 0/m$$

$$e^x y + e^x \frac{dy}{dx} + e^x \frac{dy}{dx} + e^x \frac{d^2 y}{dx^2} = -A \cos x - B \sin x \quad - - - 0/m$$

$$e^x y + 2e^x \frac{dy}{dx} + e^x \frac{d^2 y}{dx^2} = -y e^x$$

$$\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 2 = 0 \quad - - - 0/m$$

$$\therefore \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 2 = 0 \quad - - - 0/m$$

(20)

$$7 \text{ (b)(i)} \quad \frac{dy}{dx} = \left(\frac{2x+y-1}{x-2} \right)^2 \quad \dots \text{--- (i)}$$

$$\text{Let } x = X+h, y = Y+k, \quad \frac{dy}{dx} = \frac{dY}{dX}$$

$$\frac{dY}{dX} = \left(\frac{2(X+h) + (Y+k) - 1}{X+h-2} \right)^2 \quad \dots \text{--- (i) mod}$$

$$\frac{dY}{dX} = \left(\frac{2X+Y+2h+k-1}{X+h-2} \right)^2 \quad \dots \text{--- (ii)}$$

$$\text{Let } 2h+k=1 \quad \left\{ \begin{array}{l} h=2, k=-3 \\ h-2=0 \end{array} \right.$$

$$\frac{dY}{dX} = \left(\frac{2X+Y}{X} \right)^2 \quad \dots \text{--- (iii)}$$

$$\text{Let } Y = UV, \quad \frac{dY}{dX} = U + X \frac{dU}{dX} \quad \dots \text{--- (iv)}$$

Substitute eqn (iv) into eqn (iii) $\dots \text{--- (i) mod}$

$$U + X \frac{dU}{dX} = \left(\frac{2X+UX}{X} \right)^2$$

$$U + X \frac{dU}{dX} = 4 + 4U + U^2$$

$$X \frac{dU}{dX} = U^2 + 3U + 4 \quad \dots \text{--- (i) mod}$$

$$\int \frac{dU}{U^2 + 3U + 4} = \int \frac{1}{X} dX$$

$$\frac{4}{7} \left(\frac{d\theta}{\tan \theta} \right) = \int \frac{1}{X} dX \quad \dots \text{--- (i) mod}$$

$$\frac{4}{7\sqrt{7}} \theta = \ln X + C, \quad \text{but } \theta = \tan^{-1} \left(\frac{2}{\sqrt{7}} \left(U + \frac{3}{2} \right) \right)$$

(21)

$$7 \text{ (b)(i)} \frac{2}{\sqrt{7}} \tan^{-1} \left(\frac{2}{\sqrt{7}} \left(u + \frac{3}{2} \right) \right) = \ln x + C \text{ but } u = \frac{y}{x}$$

$$\frac{2}{\sqrt{7}} \tan^{-1} \left(\frac{2}{\sqrt{7}} \left[\frac{y}{x} + \frac{3}{2} \right] \right) = \ln x + C, \text{ but } x = x - 4 = x - 2$$

$u - k = u + k = u + 3$

$$\frac{2}{\sqrt{7}} \tan^{-1} \left(\frac{2}{\sqrt{7}} \left(\frac{y+3}{x-2} + \frac{3}{2} \right) \right) = \ln(x-2) + C$$

— — — O/Marks

$$(b)(ii) (1+x^2) \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} = 0$$

$$\text{Let } p = \frac{dy}{dx}, \frac{dp}{dx} = \frac{d^2 y}{dx^2} \text{ — — — O/Marks}$$

$$1+x^2 \frac{dp}{dx} = -2p = 0$$

$$\int \frac{dp}{p} = - \int \frac{2x}{1+x^2} dx \text{ — — — O/Marks}$$

$$\ln p = - \ln(1+x^2) \text{ — — — O/Marks}$$

$$p = \frac{1}{1+x^2}$$

— — — O/Marks

$$\frac{dy}{dx} = \frac{1}{1+x^2} \Rightarrow dy = \int \frac{dx}{1+x^2}$$

$$\therefore y = \tan^{-1}(x) + C \text{ — — — O/Marks}$$

(22)

7(d) From Newton's law of cooling,

$$\frac{d\theta}{dt} \propto (\theta - \theta_R)$$

$$\frac{d\theta}{dt} = k(\theta - \theta_R) \quad \text{--- o/n rule}$$

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta - \theta_R} = \int_0^t k dt$$

$$\ln[\theta - \theta_R] \Big|_{\theta_1}^{\theta_2} = [kt]_0^t \quad \text{--- o/n rule}$$

$$\ln(\theta_2 - \theta_R) - \ln(\theta_1 - \theta_R) = kt$$

$$\ln \left(\frac{\theta_2 - \theta_R}{\theta_1 - \theta_R} \right) = kt$$

$$\frac{\theta_2 - \theta_R}{\theta_1 - \theta_R} = e^{kt} \quad \text{--- o/n rule}$$

$$\theta_2 - \theta_R = (\theta_1 - \theta_R) e^{kt}$$

$$\theta_2 = \theta_R + (\theta_1 - \theta_R) e^{kt}$$

But $\theta_2 = T$ temp after time t .

$\theta_R = A$ Surrounding temp

$\theta_1 - \theta_R = B$ temp diff

$$\therefore T = A + B e^{kt}$$

(23)



(a)(i) Directrix

Is a fixed line whose distance from the focus is fixed. - (0/1 mark)

(ii) Latus Rectum

Is a focal chord perpendicular to the axis and parallel to the directrix. - (0/1 mark)

(iii) Focal chord

Is a line passing through focus and joining two points of the curve. - (0/1 mark)

(iv) Eccentricity.

Is a constant ratio of the distance of the variable point from the focus to the distance of the variable point from the directrix. - (0/1 mark)

(b)(i) $(y+5x)(y-5x) = 0$, $y^2 - x = 0$.

$y^2 = x$, the value of $a = \frac{1}{4}$. - (0/1 mark)

$y^2 = 4ax \Rightarrow a = \frac{1}{4}$

Vertex $(0,0)$

Focus $S(a,0) \Rightarrow S(\frac{1}{4},0)$

Equation of directrix $x = -a$, $x = -\frac{1}{4}$. - (0/1 mark)

(ii) $x^2 - 4y + 12 = 0$

$x^2 = 4y - 12$. - (0/1 mark)

$x^2 = 4(y-3)$ by comparing with the standard equation $(x-h)^2 = 4b(y-k)$, $h=0$, $k=3$, $b=1$. - (0/1 mark)

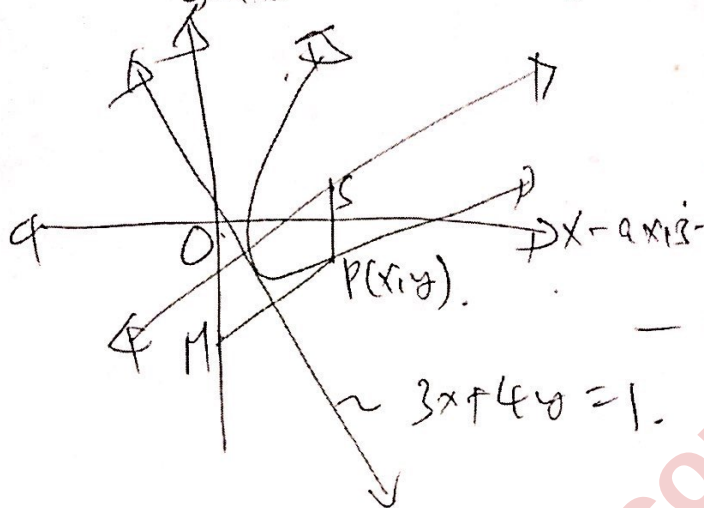
(24)

Vertex $V(h, k) = V(0, 3)$

Focus $S(h, b+k) = S(0, 4)$

Equation of directrix $y = -b+k, y = 2$
 - 01 mark

C (P) Consider the Diagram



From the figure above $\frac{SP}{MP} = e$, but $e = 1$
 - 01 mark

$SP = MP$

$\sqrt{(x-3)^2 + y^2} = \left| \frac{3x + 4y - 1}{\sqrt{3^2 + 4^2}} \right|$ - 01 mark

Sq on both sides.

$(x-3)^2 + y^2 = \frac{(3x + 4y - 1)^2}{25}$

$25[x^2 + y^2 - 6x + 9] = 9x^2 + 16y^2 + 24xy - 6x - 8y + 1$ - 01 mark

$16x^2 + 9y^2 - 144x + 8y - 24xy + 224 = 0$

$\therefore$ The equation of the parabola is

$16x^2 + 9y^2 - 144x + 8y - 24xy + 224 = 0$
 - 01 mark

(25)

8(c)(ii) Eqn of parabola is

$$x - ty + at^2 = 0$$

Given $y^2 = 4x$, $a = 1$

$$x - ty + t^2 = 0$$

Alternative

$$y^2 = 4x, \quad 2y \frac{dy}{dx} = 4$$

$$\frac{dy}{dx} = \frac{2}{y}; \quad \text{Let } \frac{dy}{dx} = M$$

Slope of Tangent at $(t^2, 2t)$; $M = \frac{2}{2t} = \frac{1}{t}$

Eqn of Tangent, $y - y_0 = M(x - x_0) + y_0$

$$y = \frac{1}{t}(x - t^2) + 2t$$

The Eqn of the Tangent is $x - ty + t^2 = 0$

(d) Given $\frac{1}{4}(x^2 + y^2 - 4y)^2 = x^2 + y^2$

$$(x^2 + y^2 - 4y)^2 = 4(x^2 + y^2)$$

$$x^2 + y^2 - 4y = 2\sqrt{x^2 + y^2}$$

$$x^2 + y^2 = 4y + 2\sqrt{x^2 + y^2}$$

but $x^2 + y^2 = r^2$, $r = \sqrt{x^2 + y^2}$

$$r^2 = 4y + 2r \quad y = r \sin \theta$$

$$r^2 = 4r \sin \theta + 2r$$

$$r = 4 \sin \theta + 2$$

$\therefore r = 4 \sin \theta + 2$

(26)

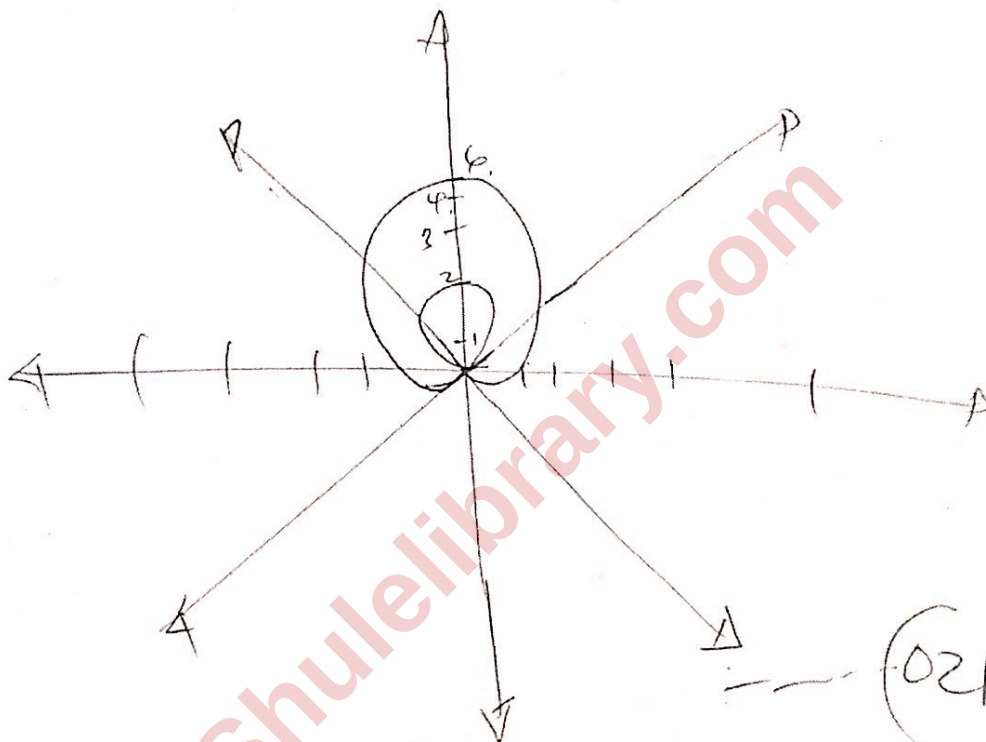
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Consider the Table below

θ	0°	30	60	90	120	150	180	210	240	270	300	330
$r = 4 \sin \theta + 2$	2	4	$2\sqrt{3} + 2$	6	5.46	5.06	2	0	-1.46	-2	-1.46	2

--- (01 Mark)

Consider the Station below.



--- (02 Mark)

(27)