THE UNITED REPUBLIC OF TANZANIA

REGIONAL ADMINISTRATION AND LOCAL GOVERNMENT

FORM SIX JOINT EXAMINATION,

ADVANCED MATHEMATICS 2

Code: 142/2

Time: 3 Hours **Tuesday, 03rd November ,2020**

Instructions

1. This paper consists of sections A and B with a total of **eight (8)** Questions.
2. Answer **all** questions in section A and **two (2)** questions from section B.
3. Sections A carries **sixty (60)** and section B carries **forty (40)** marks.
4. **All** necessary workings and answers for each question done must be shown clearly.
5. Mathematical tables and non-programmable calculators may be used.
6. Cellular phones and any unauthorized materials are **not** allowed in the examination room.
7. Write your **Examination Number** on every page of your answer sheet(s).

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SECTION A (**60 Marks**)

Answer **all** questions in this section.

1. a) i. Given that the events A and B are such that P (A) = $\frac{1}{2,}$ P (A∪ B) = $\frac{3}{5}$ and P(B) =Y_1 if they are mutually exclusive and P(B)=Y_2 if they are independent . Find the value of Y_1+Y_2

   ii. A box contains nine correction pen, numbered from one to nine. A correction pen is Drawn the box.If the number even , then a coin is tossed. If the number is prime,then a die is thrown. How many outcomes are possible**?**

   b) A family of 6 people seat on a round table for dinner.In how many ways can it be done ? if

      i. father and mother next to each other

      ii. father and mother not next to each other

   c) Given that the probability that an individual is suffering from Malaria is 0.001. Determine the probability that out of 2000 individuals at least 2 well suffer

2. (a) Write the contra positive and converse of the following logical statements

   i. "you cannot comprehend geometry if you do not know how to reason deductively

   ii. If two integers a and b are such that a is greater than b,then a-b is always a positive integer

   iii. If a quadrateral is rectangle, then it has two pairs of parallel sides

   (b) (i) Use a truth table to determine whether the following statement is valid or not

   "If I work hard, then I will get praise. If I get raise then I will not have to get a second job. I got a second Job.Therefore,I did not work hard"

   (ii) Use laws of Algebra to simplify the $[(p \wedge q) \vee [(-r \wedge (q \wedge p))]]$ and then use the results to draw an electrical network corresponding to the proposition.

   (c) A statement J is formed from P,G and M write J in most simplified form and use It to draw an electric circuit which will allow the current to flow whenever J is true

3. (a) (i) A force of magnitude 10 moves an object from A(1,2,3) to point B(3,4,5).If the force acts on the direction of the vector a=2i-2j+k,Find the work done by force

   (b) If $\Phi(x, y, z) = x^2 yz$ and a = $3x^2 yi + yzj - xzk$ find $\frac{\partial^2}{\partial y \partial z}(\Phi.a)$ at the point (1,-2,-1)

   (c) If θ is the angle between two point unit vectors a and b.show that $\frac{1}{2}(a+b) = \cos\left(\frac{\theta}{2}\right)$

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(d) a particle travels so that its acceleration is given by a= $2\,2e^{-t}i+5\cos tj-3\sin tk$ .if the particle is located at (1,-3,20 at time t=o and is moving with a velocity given by 4i-3j+2k

4. (a).Show that the roots of the equation $x^2-2x\cos\theta+1=0$ are $\cos\theta$ +isinθ and $\cos\theta$ -isinθ

(b) Given the complex number Z= $\sin\left(\frac{\pi}{5}\right)+i\left(1-\cos\left(\frac{\pi}{5}\right)\right)$, Find its modulus and argument.

Express your answer with π

(c) If $\frac{(3+4i)(1+2i)}{1+3i}$ =q(1+i), $q\varepsilon$ R.Find the value of q, Hence simplify

$\arctan\left(\frac{4}{3}\right)+\arctan 2-\arctan 3$ And leave your answer in π

(d) If imaginary part of $\frac{2z+1}{iz+1}$ is -2,where Z is a complex number then show that the locus of the point representing z is a straight line.

SECTION B

Answer **Two (2)** question from this section.

5. (a) If $\tan\alpha=\frac{m}{m+1}$, and $\tan\beta=\frac{1}{2m+1}$, then find the value of $\alpha+\beta$

(b) Given $24\cos\left(\frac{\pi}{3}x\right)+10\sin\left(\frac{\pi}{3}x\right)=13$,without using t-formula, show that x= $(6n\pm1)+\frac{3}{n}\tan^{-1}\left(\frac{5}{12}\right)$ where n is an integer

(c) (i) Find the maximum and minimum value of $\frac{1}{2\cos\theta+5\sin\theta+7}$

(ii) Prove that $\frac{\tan x+\sec x-1}{\tan x-\sec x+1}=\frac{1+\sin x}{\cos x}$

6. (a)(i) If $a^x=\left(\frac{a}{k}\right)^y=k^m$,where a ≠ 1,show that y= $\frac{mx}{m-x}$

(ii) Express $\frac{8x^2-x-1}{(x-1)^2}$ in partial fraction

(b) (i) Find the sum of the first n terms of the series $2\times3+3\times4+4\times5+\ldots\ldots$

(ii) Find the sum of the first n terms of the series $\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots\ldots\ldots$ and hence evaluate sum to infinity of the series

(c) (i) By using mathematical induction, prove that $2^{n+2}+3^{2n+1}$ is divisible by 7 for all positive integers

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(ii) Solve $\frac{2x^2-7x-4}{3x^2-14x+11} \le 2$

7. (a) Form the Differential equation from y= $(A\cos x + B\sin x)e^{-x}$

(b)(i)Solve $\frac{dy}{dx} = \left(\frac{2x+y-1}{x-2}\right)^2$

(ii) Solve the Differential equation $(x+1)\frac{dy}{dx} - y = e^x(1+x)^2$

(c) (i) Given that $(1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$,show that y= $\tan^{-1} x$ +c

(d) Suppose the initial temperature of the body was θ_1 and after time‘t’ the temperature of the body is θ_2 .Prove that T=A+B e^{-kt} ,where T is the temperature of the body after time ‘t’,A is the temperature of the surrounding (constant) and B is the temperature difference between the body and the surrounding

8. (a) Define the following
   i. Directix
   ii. Latus Rectum
   iii. Focal chord
   iv. Eccentricity

(b) Find the coordinates of the vertex, focus and the equation of directix of each of the following (i) $(y+\sqrt{x})(y-\sqrt{x}) = 0$ (ii) $x^2 - 4y + 12 = 0$

(c) Find the equation of the parabola whose focus is (3,0) and directix is 3x+4y=1

(d) Change the Cartesian equation $\frac{1}{4}(x^2+y^2-4y)^2 = x^2+y^2$ into polar equation and then sketch the corresponding curve of the equation above.

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