Arusha Regional Terminal Examination December 2022

PRESIDENT'S OFFICE
REGIONAL ADMINISTRATION AND LOCAL GOVERNMENT
ARUSHA REGION
FORM FIVE TERMINAL EXAMINATION
ADVANCED MATHEMATICS

Time: 3:00 Hours **29th NOVEMBER 2022**

Instructions

1. This paper consists of **ten (10)** questions each carrying **ten (10)** marks.
2. Answer **ALL** questions.
3. All necessary working and answers of each question done must be shown clearly.
4. NECTA's Mathematical tables and non-programmable calculators may be used.
5. Cellular phones and any authorized materials are **not** allowed in the examination room.

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1. Use a non-programmable calculator to:
   a) Calculate
      i. $\left(\frac{5.62\ln(321.62)}{e^{1.2942}\sqrt{\log_3 8}}\right)^{\frac{1}{5}}$ correct to three decimal places.
      ii. $\sqrt[5]{\frac{5\ln 4+\cos^{-1}(0.672)}{1+\sin^2(54°)}}+\sqrt[3]{\frac{\pi}{\frac{22}{7}+\pi^2}}$ correct to four significant figures.
   b) Evaluate $\sum_{x=1}^{5} {}^{10}C_x(0.6)^x(0.4)^{10-x}$

2. a) Use appropriate laws of set,
      i. Prove that $(A \cap B) \cup A = A$. ii. Simplify $A - (A - B)$.
   b) One of poultry farm in Arusha which produces three types of chicks had its six-months report which revealed that out of 126 of its regular customers, 65 bought broiler, 80 bought layers and 75 bought cocks. 45 bought layers and cocks, 35 bought broilers and cocks, 10 bought broilers only, 15 brought layers only and 6 bought cocks only. Six (6) of the customers did not show up.
      i. How many customers bought all three products?
      ii. How many customers exactly bought two of the farms' products**?**

3. (a) Use algebraic law of set, simplify $[(A - B) \cup (A - C)] \cap A'$
   (b) Given $A = \{x: x\epsilon\ \Re\ |x| \geq 2\}$, and $B = \{x: x\epsilon\ \Re\ \ 1 \leq x < 6\}$ use number line representing $(A \cap B')$

c) The research at certain company was conducted for the 6 workers to either accountants Auditor or both, the number of Accountants exceed the number of Auditor by 1 and the product of the number of Auditors only to that of Accountant only is equal total of all workers. How many workers are Accountants?

4. (a) Find the area of triangle formed by line $2x - 3y = 12$ and axis.

(b) A point P moves so that it is equidistant from the point A(1,2) and B(−2, −2),find the cartesian equation of the locus P.

(c) Show that the common chord of two circles $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ is perpendicular to the line joining their centers.

5. (a) Find the equations of tangent to the circles that has radius 4 and center at origin, drawn from the point (1,4).

(b) If P is the perpendicular distance from the origin of the line whose intercept on the axes are **a** and **b**, show that $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}$

(c) Find the ratio which the line $3x - 2y + 5 = 0$, divide the line joining the points (6, −7) and (−2,3).

6. (a) If $f(x) = 1 - 3x - 5x^2$ and $g(x) = x^2 + 2x$ find coefficient of x^2 from fog(x).

(b) Function f(x) defined by $f(x) = \frac{x(x-5)}{(x-3)(x-2)}$

i. What is intercepts of **f**(x).
ii. Determine vertical and horizontal asymptotes of **f**(x) .
iii. Sketch graph of f(x).

7. a) If $f(x) = \frac{3+x^2+x}{x+1}$ and $g(x) = \frac{-4}{x}$ when $x \neq 0$ find the values of x if $fog(x) = -5$

b) Given $f(x) = \begin{cases} 7x^2 + x - 8 & x \leq 1 \\ 4x + 5 & 1 < x \leq 7 \\ 8x + 3 & x > 7 \end{cases}$ and

$g(x) = \begin{cases} |x| & x < -3 \\ 0 & -3 \leq x < 2 \\ x^2 + 4 & x \geq 2 \end{cases}$

Find values of fog(−3) and fog(5)

c) Given that $f(x) = x^4 - 2x^3 - x^2 + 2x$

(i) Find the values of **x** where the curve **f(x)** cuts the **x** −axis.
(ii) Sketch the graph of **f(x)**

8. a) i. By using algebraic law of preposition show that $[(p \rightarrow q) \wedge p] \rightarrow q$ is tautology.

ii. Determine the contrapositive of proposition "If *x* is less than zero then *x* is not positive"

b) Use the truth table to test the validity of the following argument "if a am intelligent, then I will pass this examination. I am intelligent. Therefore, I pass this examination"

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c) Let p be the proposition "Myra reads English" and q be the proposition "Myra reads Kiswahili" and r be the preposition "Myra reads French"
Write each of the following proposition in symbolic form

i. It is not true that Myra reads English or French.

ii. Myra reads English but not Kiswahili.

9. a) i) Define term logical equivalence.

ii) Show that $(p \leftrightarrow q)$ is logical equivalence to $\sim (p \curlyvee q) \curlywedge (p \curlyvee \sim q)$

b) Test the validity of argument;
"If I study hard, then I will not fail Mathematics. If I do not fail to manage my time then I will study hard. But I failed Mathematics. Therefore, I failed to manage my time"

c) Consider the truth table below

p	q	S(p,q)
T	T	T
T	F	F
F	T	T
F	F	T

i. Write the compound statement equivalent to the truth table (S)

ii. Simplify the compound statement for (S)

iii. Draw the corresponding network circuit of (ii)

10. (a) Solve the following simultaneous equations

$$\log(x + y) = 0$$
$$2\log x = \log(y + 1)$$

b) (i) Find the sum of the first n terms of the series $3 \times 4 + 5 \times 3 + 7 \times 2 + 9 \times 1 + \cdots$

(ii) If α and β are the root of quadratic equation $2x^2 - 8x + 5 = 0$, find value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

c) (i) State principle of mathematical induction.

(ii) Prove that $\mathbf{x^n - y^n}$ has $\mathbf{x - y}$ as factor for all positive integers n by mathematical induction.

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